The Correct Qibla

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(Last Revised 1997/9/17)†

1 Introduction

A book[21] published recently by Nachef and Kadi argues that for North America the qibla (i.e., the direction of Mecca) is to the southeast. As proof of this claim, they quote from a number of classical Islamic jurisprudents. In further support of their view, they append testimonials from several living Muslim religious scholars as well as from several Canadian and US scientists. The consulted scientists—mainly geographers—suggest that the qibla should be identified with the rhumb line to Mecca, which is in the southeastern quadrant for most of North America. The qibla adopted by Nachef and Kadi (referred to as N&K in the sequel) is one of the eight directions N, NE, E, SE, S, SW, W, and NW, depending on whether the place whose qibla is desired is situated relatively east or west and north or south of Mecca; this direction is not the same as the rhumb line from the place to Mecca, but the two directions lie in the same quadrant.

In their preliminary remarks, N&K state that North American Muslim communities used the southeast direction for the qibla without exception until the publication of a book[1] about 20 years ago. N&K imply that the use of the great circle for computing the qibla, which generally results in a direction in the northeastern quadrant for North America, is a new idea, somehow original with that book. They find the idea a source of grave wrongdoing, invalidating the prayers of many North American Muslims.

I have no religious expertise and take no interest in religious controversies. Had N&K's criticism of [1] been based only on religious rulings, I would not have responded to their book. But its arguments touch on scientific subjects. Even more seriously, some scientific professionals have provided what looks like a scientific seal of approval to incorrect opinions.

Many of the simpleminded ideas discussed in [21] by the authors and their correspondents were examined and dismissed by Muslim astronomers centuries ago. These ideas are a consequence of thinking in terms of a flat map of the earth, inferring relationships from it that are false on the actual globe. The source of much confusion is the deceptive mindset that the meridians (north-south directions) are mutually parallel. The fact is that, for good reasons, Muslim astronomers have been using the great circle direction for determining the qibla for the last twelve centuries. And contrary to what N&K claim, mosques oriented in the great circle direction to Mecca were in existence in North America at least as early as 1953, 25 years prior to the publication of [1].

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†PDF version in `http://www.patriot.net/users/abdali/ftp/qibla.pdf`

This paper attempts to clarify several common misunderstandings about the qibla. It explains the basic concepts underlying the qibla problem, in particular, the great circle and the rhumb line. It then gives the reasons why the great circle gives the correct direction, and the rhumb line or the quadrant idea gives the incorrect one, for determining the qibla. The paper also describes methods for the computation of qibla. Detailed comments on N&K's book are given separately in an appendix. The discussion is confined to the scientific aspects of the subject, and religious rulings are analyzed only for their underlying scientific assumptions.

2 Historical Background

The book[21] begins with a "Historical Precis on the Subject" as follows:

> The Muslims came to North America at the beginning of the previous *hijriyy* century.... It was only natural for them to endeavor to establish mosques and *musallayat* to gather in for performing congregational prayers... . Based on the *fatawa* of the scholars of the Muslim countries, all of the *maharib* of these mosques were facing southeast without exception. Some twenty years ago, and specifically between 1973 and 1974, *ˆAbdullah al-ˆAbdaliyy* published a book in which he mentioned that the direction of *al-Qiblah* in the USA and Canada is northeast. ... In North America, those who negate *at-tawassul* along with members of the party known as *al-'Ikhwan al-Muslimin* were impressed with this book and adopted it, as is their usual habit with deviant matters. Par to course, they spread and promoted that book with all that it contains which fragments the unity, divides the word of the Muslims, and perverts the Religion. ... Some of us, in person, spoke with *ˆAbdullah al-ˆAbdaliyy*, the author of the book, and asked him about the religious evidences he relied on in producing his book. His answer was: "I did not study religious studies. However, I have relied on the spherical triangular calculations."

The only correct statement in this quotation is the paraphrase of my statement that I did not go through a religious curriculum and that I used spherical trigonometry in computing the qibla. Other than that, the above is a grossly inaccurate account, with errors in facts, names, and dates. I infer that N&K are referring to the book *Prayer Schedules for North America* which was published in 1978[1], not in 1973. The author's name is not ˆAbdullah al-ˆAbdaliyy but S. Kamal Abdali. Tawassul is irrelevant to the topic, and the Ikhwan's implication is unwarranted and baseless. The qibla computation section [1, pp. 289–292] lists three references to prior work by others, so there is no justification for blaming that book for starting what is later called a new "deviant endeavor."

2.1 North American Practice

A serious misstatement in the above account is that all the mosques in North America ("*without exception!*") were oriented toward southeast, and that the northeast direction was adopted only after the publication of the above book in 1978 (1973-74 according to N&K). This is incorrect. Of many mosques built much earlier with a northeastern orientation,[1] the case of the Islamic Center of Washington, D.C., built in 1953, is well documented. Here is an excerpt from *The Washington Daily News*, April 15, 1953, reprinted in *Surveying and Mapping*[19]:

> THE MOSQUE on Massachusetts Avenue [in Washington] faces 56 degrees, 33 minutes, and 15 seconds east of true north, and thereby hangs a minor tale.

[1] For brevity, I use "northeast" to refer to generally any direction in the northeastern quadrant, not just $45°$ E of N. Similarly for other quadrants.

> The direction is toward Mecca, which all mosques must face with mathematical exactness. It was calculated for this mosque by the Egyptian Ministry of Works in Cairo. . . .
>
> Passersby—many of them Moslems—keep dropping in, and some of them have come up with a frightening thought. The direction 56-33-15, they pointed out, was slightly north of east. But Mecca, the Moslem holy city in Arabia, was south of Washington. . . . Take any map, they said, and draw the line between the two cities and the line would go slightly south. . . .
>
> The problem is easily explained. The visiting Moslems had been thinking in terms of the standard Mercator projection map. On such a map the straight line *would* go slightly south, but it would not be the shortest distance to Mecca. The shortest distance, as an air age knows, would be a great circle route, easily seen on a globe or on a special flat map called a "gnomonic projection"... It was like the great circle route suddenly coming to the Moslem world—except, of course, that Moslems drawing circles in the sand centuries ago had known about it all along.

The newspaper story goes on to describe how the Egyptian Ambassador, who was the president of the organization building the mosque, was unsure about this direction, and questioned the architect Irwin S. Porter about it. The story then continues:

> "I'll tell you, it gave me a few nights of worry," Mr. Porter said.
>
> Mr. Porter called up a cartographer, Wellman Chamberlin, of the National Geographic Society, and asked him "just for formality" to check the direction of Mecca.
>
> "He called back in a few hours," Mr. Porter said, "and of course it was right—56-33-15."

2.2 History of Qibla Computations

To return to N&K, their "Historical Precis" continues with this conclusion[21, p. 8]:

> . . . Sadly, the matter of determining the direction of *al-Qiblah* has become now-a-days a play thing between the hands of he who does not have the religious knowledge and wants to occupy his leisure time.

N&K also seem unaware that the fascination with qibla computation is not something new. The problem has received serious attention from some of the most famous Muslim scientists, including al-Khwarizmi (780–850), al-Battani (858–929), Abu al-Wafa al-Buzjani (940–997), Ibn al-Haitham (965–1040), al-Biruni (973–1048), and al-Tusi (1201–1274). This is a veritable who's who of medieval science.[2] Significant original contributions to the qibla determination were also made, among many others, by Habash al-Hasib (*ca.* 850), al-Nayrizi (*ca.* 897), Ibn Yunus (*ca.* 985), al-Khalili (*ca.* 1365), and Ibn al-Shatir (1306–1375). Though

[2] Al-Khwarizmi's greatly influential treatise on algebra gave the name to that discipline. The word *algorithm* derived from his name has become a household word with the spread of computers. Al-Battani determined several astronomical quantities with remarkable precision. In trigonometry, he introduced new laws, new functions, and new formulations of old functions. Abu al-Wafa made significant contributions to mathematics and astronomy. He interrelated the six main trigonometric functions. He is, arguably, credited with discovering a component of the moon's motion, rediscovered 600 years later by Tycho Brahe. Ibn al-Haitham, famous for his foundational work on optics, also worked prolifically in mathematics and astronomy. He was a pioneer of the scientific method consisting of observation, hypothesis formulation, deduction, and experimental verification. Al-Biruni is described by Durant[11, p. 243] thus: "Philosopher, historian, traveler, geographer, linguist, mathematician, astronomer, poet, and physicist—and doing major and original work in all these fields—he was at least the Leibnitz, almost the Leonardo, of Islam." Al-Tusi systematized trigonometry (plane and spherical) as a discipline independent of astronomy. He also formulated a non-Ptolemaic model of planetary motion based on spheres.

perhaps less well-recognized than the persons in the first list, several of these people have done outstanding work in astronomy and mathematics.[3,4]

Some of the most important early work on determining the qibla is the following: Approximate methods were given by Al-Khwarizmi and Al-Battani. Due to the simplicity of its geometric construction, Al-Battani's method remained in wide use even after more accurate methods became available.[5] Exact methods based on graphical constructions were given by Habash al-Hasib and Ibn al-Haitham, and those based on such constructions and spherical trigonometric computations were given by Ibn Yunus, al-Nayrizi, and al-Biruni. Tables containing the qibla angle as a function of longitude difference from Mecca and latitude were compiled by Ibn Yunus and al-Khalili. In addition to these, practical methods involving astronomical instruments such as astrolabes and various types of quadrants were devised by numerous researchers, many of them unidentified. The qibla could be determined by solar observations directly at certain times, and derived from observations using spherical trigonometric calculations at other times. The versatile astrolabe served well both for performing astronomical observations and for practically solving spherical triangles without trigonometric tables or the labor of arithmetic.

The history of scientific qibla determination has been the subject of extensive research, most notably by Schoy, Kennedy, and King. The reader is referred to King [16, sections IX and XIV] for papers on the subject. Several methods have been mathematically analyzed by Berggren (e.g., [6]). An Arabic reference mentioning some early work is [15].

The ignorance and neglect of centuries of outstanding Muslim scientific work on the qibla problem are truly distressing. It therefore seems appropriate here to include a rather long quotation from King [16, pp. I:253–258], summarizing the history of the subject:

> Muslim astronomers from the eighth century [A.D.] onwards concerned themselves with the determination of the qibla as a problem of mathematical geography. This activity involves the measurement of geographical coordinates and the computation of the direction of one locality from another by procedures of geometry or trigonometry. The qibla at any locality was defined as *the direction to Mecca along the great-circle on the terrestrial sphere.* [my italics]
>
> ... Already in the early ninth century observations were conducted in order to measure the coordinates of Mecca and Baghdad as accurately as possible, with the express intention of computing the qibla at Baghdad. Indeed, the need to determine the qibla in different locations inspired much of the activity of the Muslim geographers. The most important Muslim contribution to mathematical geography was a treatise by the eleventh-century scientist al-Biruni. His purpose was to determine for his patron the qibla at Ghazna (in what is now Afghanistan), a goal which he achieved most admirably.
>
> Once the geographical data are available, a mathematical procedure is necessary to determine

[3] Habash al-Hasib introduced new trigonometric functions. Ibn Yunus compiled very accurate astronomical tables, studied the motion of the pendulum leading to the invention of mechanical clocks, and invented several astronomical instruments. Ibn al-Shatir constructed a model of planetary movements which, though geocentric, was mathematically identical to the model given by Copernicus 200 years later.

[4] A single source for reading about the life and scientific contributions of the above-mentioned Muslim scientists, is [10]. For their contributions specifically to the qibla determination problem, a recommended starting point is [16]. In spite of King's own prolific work and many other references cited by him, he laments [16, p. IX:16] that numerous Islamic astronomical works containing qibla computations still remain uninvestigated.

[5] Al-Battani's construction, described in [5, pp. 40–41], [16, pp. 103–107], and [18, p. 360], was included in *Mulakhkhas fi al-Hai'a* (ca. 1325), al-Jaghmini's textbook on astronomy. A commentary on this book, popularly called *Sharh Chaghmini*, has been part of the standard curriculum of the subcontinental Islamic colleges even in the 20th century. As the *Sharh* points out (per Bannuri[5, p. 40]), al-Battani's method does not work (the construction does not make sense) for places whose longitude differs from that of Mecca by 90° or more. The method is thus not applicable to North America. But if brute-forced anyway, the construction will yield a southeast direction for the qibla of most North American locations. This would be the charitable way of explaining the ruling from Bareilly[21, p. 58].

the qibla. The earliest Muslim astronomers who considered this problem developed a series of approximate solutions, all adequate for most practical purposes, but in the early ninth century, if not before, an accurate solution by solid trigonometry was formulated.. . .

Over the centuries, numerous Muslim scientists discussed the qibla problem, presenting solutions by spherical trigonometry, or reducing the three-dimensional situation to two dimensions and solving by geometry or plane trigonometry. They also formulated solutions using calculating devices. But one of the finest medieval mathematical solutions to the qibla problem was reached in fourteenth-century Damascus: a table by al-Khalili displays the qibla for each degree of latitude from 10° to 56° and each degree of longitude from 1° to 60° east or west of Mecca, with entries correctly computed according to the accurate formula. . .

2.3 Great Circle Computations of the Qibla

While the classical scientific works on the qibla problem may be difficult to access, there are dozens of modern books about it—written independently in different countries and different languages. All the ones that I am aware of use the great circle definition opposed by N&K. Representatives of this literature are [2, 3, 4, 13, 12, 14, 18]. Some of these books have been written by religious scholars, and some have been endorsed by them. The author of the Urdu book[18] is himself a religious scholar and a *mufti*. The Urdu book[14] includes a foreword and several endorsement letters by well-known religious scholars of Pakistan. The endorsers include Muhammad Shafī‘ and Yūsuf Bannūri, both of whom were very distinguished *mufti*s and were Principals of well-known religious colleges in Karachi. The Farsi book [2], which tabulates the qibla angle for most of the well-known cities in the world, contains a foreword by Shaikh Hussain Khurāsāni, with the title *Hujjat al-Islām* reserved in Iran for Shiite religious scholars of high standing. Bannuri, mentioned above, himself wrote a most comprehensive book[5] about the jurisprudential aspects of the problem. This book also includes a discussion of the scientific aspects, and has a section that gives the great circle definition for the qibla and describes eight methods based variously on geometric construction, solar observation, and use of astronomical instruments. At age 23 (when he wrote the book, see p. 166), the author seems traditional and indifferent toward astronomical methods. Still, according to him, the use of geometric and astronomical methods for determining the qibla is permissible, though not required. Thirty years later, he appears to have become more sympathetic toward these methods as implied by his endorsement article in [14].

While many theologians have ignored, discouraged, or opposed scientific investigation of the qibla problem, there are several eminent exceptions. N&K[21, p. 15] refer to the *Tafsir* by al-Razi (1149–1204) for his sayings about finding the north direction. In the same classic work, al-Razi also discusses the qibla problem in great detail[20, vol. 4, pp. 123–138]. When describing "exact" methods [pp. 131–132] for determining the qibla, he gives its great circle definition, and describes how to find it with the aid of an astrolabe. This method (also quoted in [5]) is a way of solving spherical triangles on the astrolabe to get the time of the day when the sun is in the same direction as the qibla. The result of using the method narrated by al-Razi is equivalent to using the trigonometric formula that N&K find "deviant."

2.4 Responses from Scientists

N&K introduce the scientific testimonials included in their book in this way[21, p. 59]:

> We reproduce the replies of some astronomers, geographers, universities, geographic and marine institutions in [USA and Canada] and others regarding the shape of the earth as well as the position of *Makkah* with respect to North America.
>
> All these show that aside from negating the methodology of the scholars of *al-'Islam*, those who claim *al-Qiblah* in North America is toward northeast are also negating the sayings and

> methodologies of the current Westerners who work with the science of astronomy and geography.

Unaware that there is a long tradition of Muslim scientific research on the qibla, N&K imply that the use of great circles for qibla computation in [1] has resulted from its author's 1) ignorance of the religious aspect of the problem and 2) superficial acquaintance with "western" science. They have not seen it fit to consult any Muslim astronomers or geographers to get an alternate opinion. Instead, they have sought answers from the "current westerners" most of whom have been clearly caught off their guard, and would probably have answered differently had they been asked the correct question or had they been aware of the background scientific research spanning 12 centuries.

Some of the answers are plainly wrong, such as Klinkenberg's statement[21, p. 66] that "If a tower were to be built in Mecca that could be viewed from North America, that tower would appear on the East Southeast horizon." As proved in Section 3.5 using elementary geometry, the tower can only appear in the great circle direction, northeast from wherever in North America it can be seen. Another erroneous statement is by Frost[21, p. 71]: "…[In Montreal] observe the location of the sun at noon (12:00) Eastern Standard Time. A vertical stick will cast a shadow that is exactly N-S…" His reason is that "Montreal is located at 75° W, the central meridian of Eastern Standard Time." Noon, the time of sun's meridian transit and the time when shadows point north-south, should not be confused with 12:00. All that Montreal's special location does is to make its standard time equal to its local time. But the local time of noon varies from day to day (a table is given, for example, in [1, p. 288]) and is seldom the same as 12:00. Moreover, suggesting the use of noontime shadows to determine the north direction is bad scientific advice. Noontime shadows are very short, and extending short lines introduces relatively large angular errors.

Some of the answers are inaccurate or imprecise, such as Ryan's statement[21, p. 61] that "the great circle route is not a straight line, but an arc.… By contrast, a straight line from Washington, D.C. to Mecca follows a straight path in an east southeast direction." This statement is misleading because it fails to mention that it is describing the situation only on one particular representation of the earth's spherical surface on a plane, namely, the Mercator projection map. On the earth's surface itself there is no such thing as a straight path. The great circle is the closest thing to a straight line there can be on a sphere. On the other hand, the east southeast "straight line" she mentions is the rhumb line which is a spiral path on the actual surface of the earth.

Some of the answers are confused and misleading, such as Moore's statement[21, p. 68] that "the Great Circle route direction, with a constant bearing from Montreal will terminate at the North Pole." (Knight and Binghardt [21, p. 70, 72, resp.] make similar assertions.) If it is meant that an object launched in Montreal in the great circle direction toward Mecca, and allowed to move under its own momentum, will end up at the north pole, then the statement is false. So presumably the intention is that someone who starts from Montreal on the great circle course toward Mecca, and then maintains the initial angle from north along the way as if doing the rhumb line navigation, will reach the north pole instead of Mecca. This is true, but any navigator who so ineptly mixes great circle and rhumb line courses deserves worse! Moreover, the mention of the initial bearing as a great circle route direction is pointless in this statement since any northern bearing will have the same effect. I discuss the letters from scientists in detail in the appendix.

N&K continue[21, p. 59]:

> Hence, those deviants have not followed the methodology of the Imam's of this Religion (as it is required of them), nor have they followed the methodology of astronomers and geographers. They deviated from both. So with whom are they?

N&K are confused about the methodology of scientists. This methodology does not require following any particular person—no matter how distinguished his or her credentials—without critically examining his

or her ideas. As evident from the examples above, some of N&K's "westerners" have not represented astronomy or geography to scientific standards, and have provided answers without due reflection. It just so happens that the great circle definition of the qibla has been adopted by some of the most outstanding scientists of their time. But we accept their idea only because it makes most sense, specifically because the great circle direction best fits the notion of qibla on the round earth. We can now compute the qibla much faster and more accurately than these scientists could. But their definition of qibla has not been made obsolete by any scientific discovery made since their time, including the more precise recent knowledge about the shape of the earth gained from satellite-based images—the knowledge that in N&K's mistaken opinion has invalidated the great circle definition. I'll discuss this point again later.

I don't want to be forced to "be with" anybody. But what a joy it is to be with Ibn al-Haitham and al-Biruni! Truly rare intellects, many of their ideas, not just the ones relating to the qibla, have withstood the test of time for the past 1000 years.

3 Preliminary Notions

The qibla determination is a trivial computational problem nowadays. There is little justification for investigating new algorithms for it, and, were it not for the appearance of books such N&K's, little need to revisit the old definitions. But so much confusion has been created by N&K's book that it is best to start with basic concepts. In particular, I describe great circles and rhumb lines in much detail. I feel this is warranted since even some professional geographers that have corresponded with N&K seem to have misconceptions about these simple notions.

3.1 The earth's shape

The earth is shaped like a sphere but flattened a little at the poles and bulged a little at the equator. The diameters range from 12,714 kilometers at the poles to 12,756 kilometers at the equator (7,900 miles to 7,926 miles) approximately. The smallest and the largest values of diameters thus differ by about one part in 300. The deviations from the spherical shape due to mountains and canyons, which measure less than 10 kilometers (6 miles), are even more negligible as they cause the extreme diameter values to differ by less than one part in 1,500. For the purpose of qibla determination, the earth can be taken to be a perfect sphere, as this assumption does not affect the accuracy of the result in a practically significant way, and not any more than the uncertainties in the geographical coordinates used in the calculation.

3.2 Great circles, antipodes

Two points on a sphere are called each other's *antipodes* when they lie at the two ends of a diameter. For example, the earth's north and south poles form a pair of antipodes.[6] If a plane cuts a sphere, the curve of intersection on the sphere is always a circle. If the plane cutting the sphere also passes through the sphere's center, then the sphere is cut into two equal parts (hemispheres), and the circle of intersection is called a *great circle*. A great circle has the same center and radius as the sphere itself, and is the largest circle that can be drawn on the spherical surface. The intersection of a sphere and a plane not passing through the sphere's center is called a *small circle*. These concepts are illustrated in Figure 1. In the left picture, the plane p passing through the sphere's center O cuts the sphere along the curve $ABCDA$ which is a great circle. The points A and C are antipodes. The dotted lines are small circles. The solid lines on the sphere are great circles, passing through the antipodes N and S. (S cannot be seen exactly as it lies behind the visible

[6] It can be checked from an atlas that the antipode of the Ka'ba (lat. $21^\circ 25' 24''$ N, long. $39^\circ 49' 24''$E) is a point (lat. $21^\circ 25' 24''$ S, long. $140^\circ 10' 36''$W) in the Pacific Ocean in the vicinity of the Tuamoto Archipelago in French Polynesia.

part of the sphere.) The right picture shows a full, plane view of the great circle $ABCDA$, together with some other parts that are discussed later.

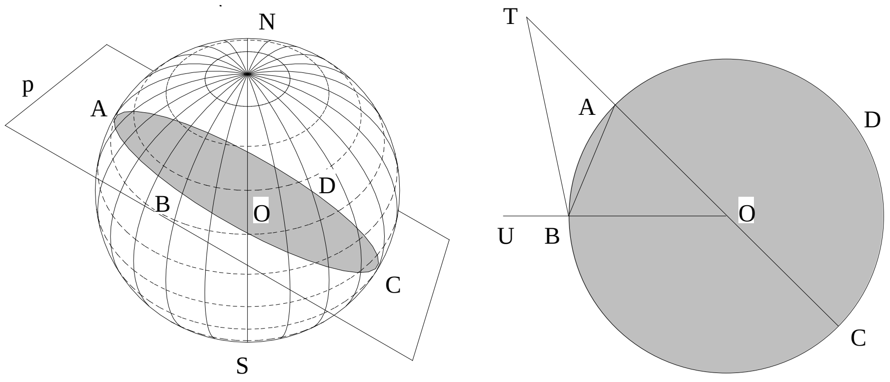

Figure 1: Great circles, small circles, and antipodes.

Since great circle arcs are curved lines, their sizes can be given in linear measure such as meters or feet. But often an angular measure is used for this purpose, specifying the size of a great circle arc by the angle between the lines from the arc's end points to the center. The distance between two points on a sphere is similarly often given by the angular measure of the shorter great circle arc between them. In Figure 1 (right), the size of the arc AB is thus specified by the measure of the angle AOB. The distance between A and B is also given by the angle AOB.

3.3 Parallels and meridians (lines of latitude and longitude)

The cause of the apparent daily motion of the sun, moon, and stars in the sky is the earth's daily rotation. The axis of this spinning movement is a diameter of the earth whose two ends are called the *north* and *south poles* (Refer to Figure 2).

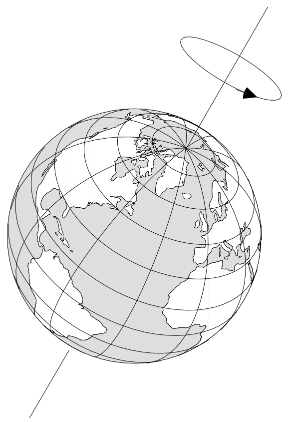

Figure 2: The spinning earth.

The two poles remain stationary while every other point on the earth's surface traces a complete circle

every 24 hours. All these circles are the intersection of the earth's surface and planes which are perpendicular to the earth's axis of rotation. These circles, which are mutually parallel in the sense that they lie in parallel planes, are called *parallels* or *lines of latitude.* (Refer to Figure 3.) Among these circles the one whose plane passes through the earth's center—and which therefore is a great circle—is called the *equator*. All parallels other than the equator are by definition small circles. The equator is the longest parallel, about 40,075 kilometers (24,900 miles) in length. Parallels become shorter and shorter as we get closer and closer to either pole. For example, the parallels passing through points half and two-thirds of the way from the equator to the pole are about 28,300 and 20,000 kilometers (17,700 and 12,500 miles) long, respectively, and the parallel passing through a point 20 kilometers (13 miles) from the pole is only about 63 kilometers (40 miles) long.

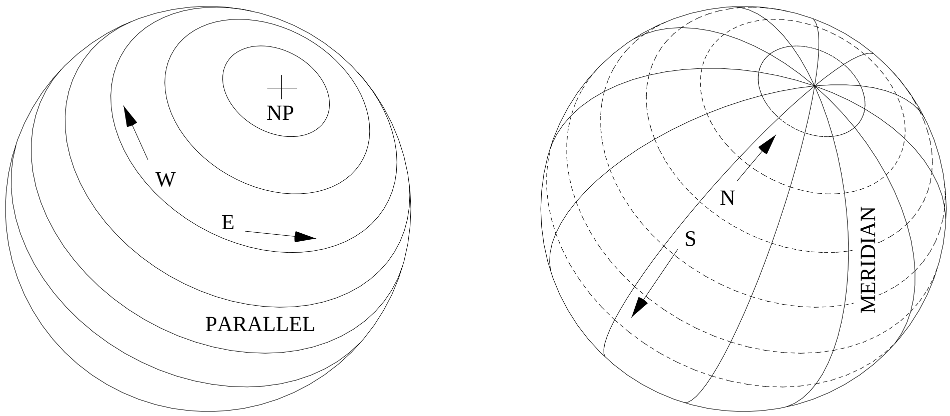

Figure 3: Left: Parallels (lines of latitude, E-W). Right: Meridians (lines of longitude, N-S).

The great circles passing through the north and south poles are called *meridians* or *lines of longitude.* Actually the term meridian refers to only a half of the great circle, extending from pole to pole. Meridians are not parallel lines as they converge on the poles. Any two meridians will be seen to be farthest apart at the equator, coming closer as we move toward either pole, and actually meeting at the poles.

3.4 Cardinal directions, latitudes, and longitudes

The earth's daily rotation is also the basis for defining the cardinal directions. We have seen that each point on the earth which is not a pole moves along the parallel passing through that point. The direction in which the point moves is called east, and the opposite direction is called west. The directions along the meridian are called north and south, depending on the pole to which they lead.

Parallels and meridians provide a unique way of specifying the location of points on the earth's surface. We have seen above that the size of great circle arcs can be given in angular measure, for example, degrees, by measuring the angle the arc spans at the sphere's center. The *latitude* of a place is measured along the meridian passing through that place, by the arc between that place and the equator. The *longitude* of a place is measured along the equator, by the arc between the meridian passing through that place and an arbitrarily chosen meridian designated as the *prime meridian.* Refer to Figure 4. The point A is situated on the parallel PAQ and the meridian NAS. $ECBF$ is the equator, NCS the prime meridian, and O the center of the earth. The meridian of A and the prime meridian intersect the equator at B and C, respectively. Then the latitude of A is the arc BA or the angle BOA, and the longitude of A is the arc CB or the angle COB.

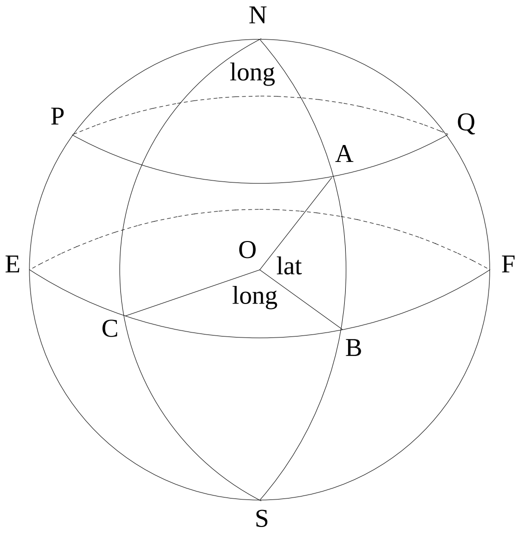

Figure 4: Latitude and longitude.

At their point of intersection, any two great circles form a *spherical angle* on the surface of the sphere, and this angle is defined to be equal to the (solid) angle between the planes of the two great circles. It is a consequence of this definition that the longitude of A is also given by the spherical angle ANC between the meridian of A, $NABS$, and the prime meridian NCS.

Latitude values range from $0°$ at the equator to $90°$ north or south at the poles. The arc AN therefore equals $90° -$ latitude of A; it is the distance of A from the North Pole, or the *colatitude* of A. Longitude values range from $0°$ at the prime meridian to $180°$ east or west. Actually, the same meridian is both $180°$E and $180°$W, and it is also the other half of the great circle containing the prime meridian. In the classical Muslim scientific works on qibla determination, the prime meridian was usually taken to be that of the western coast of Africa or of the Canary Islands, then considered the "edge of civilization." By international agreement, the prime meridian now is the meridian through Greenwich, England. The selection of prime meridian is really irrelevant to qibla computation, because, as we will see, the qibla at any place depends only on the *difference between the longitudes of Mecca and that place*, and this difference, of course, remains the same no matter where the prime meridian is.

The term *geographical coordinates* refers to a pair of values, latitude and longitude. There is a one-to-one correspondence between geographical coordinates and locations on the earth. A latitude value determines a unique parallel, and a longitude value determines a unique meridian. The intersection of these two lines determines a unique point on the earth. Conversely, each location on the earth has a unique geographical coordinate pair, determined by the meridian and parallel passing through that point.

For simplicity, we think of parallels (lines of latitude) as the lines of east-west directions also. The east-west direction at any location is, however, not the parallel itself, but the great circle which is tangent to the small circle formed by the parallel passing through that location. Meridians and these east-west great circles intersect at right angles.[7] So at each point on the earth, the east-west and north-south directions are perpendicular to each other. But it is to be emphasized that we are talking about local relationships here. If two systems of lines are drawn on a plane such that each line of the first system is perpendicular to each line

[7]This is what is meant by the commonly made statement that meridians and parallels mutually intersect at right angles. Rigorously, this simplified statement is incorrect. Spherical angles are defined only between great circle arcs. Since parallels (except the equator) are not great circles but small circles, it makes no sense to talk about angles involving parallels.

of the second system, then the lines in each system are necessarily parallel. This is not true on a sphere, as is manifest for parallels and meridians. Several properties that would have been true about cardinal directions, had the earth been flat, do not hold on the spherical earth.

The lines in space indicating the same cardinal direction (east, west, north, or south) at two points on the earth are not, in general, parallel. By looking at the meridians, we see that the north and south directions are never parallel except on the points at the equator. Since the parallels (lines of latitudes) are parallel circles, there is a misconception that east-west directions are everywhere parallel. This is false. The lines indicating the east direction, for example, are parallel only for points on a common meridian. Even for points on the same parallel, the lines indicating east are not parallel. For points on the two meridians that are halves of the same great circle, the lines are parallel but have opposite sense. For example, viewed in space, the east direction in Edmonton, Canada (long. 113°W) is the same as the west direction in Karachi, Pakistan (long. 67°E)! Another important fact is that at the poles there is but one direction. All directions are south at the north pole and north at the south pole.

The precise description of a direction is given by the angle that direction makes with one of the cardinal directions. Conventionally, the angle is measured clockwise from north. The terms *bearing*, *azimuth*, and *heading* are nowadays used synonymously to refer to this angle.[8] Thus, north, northeast, east, southeast, south and west directions have, respectively, the bearings 0°, 45°, 90°, 135°, 180°, and 270°. For convenience, directions are sometimes specified more explicitly in the form 10° E of N or 10° W of N, instead of the bearing 10° or 350°, respectively.

3.5 Properties of Great Circles

Great circles are crucial to computing the qibla, so I now state some important facts about them.

One and only one great circle passes through any two given distinct points on a sphere that are not mutual antipodes. This is so because the three non-collinear points consisting of the sphere's center and the two given points determine a unique plane, and the intersection of this plane and the sphere uniquely determines that great circle. On the other hand, infinitely many great circles connect any two antipodes, since these two points and the sphere's center are on a straight line—a diameter—and infinitely many planes contain this line.

On a sphere, the shortest path between two points is along the great circle between them. Two distinct points lying on a great circle divide the great circle into two arcs. Unless the two points are antipodes, the two arcs are of different lengths. The shorter arc is the shortest distance on the sphere between the two points. Proving this assertion is difficult, but intuitively it seems as evident as the statement that the shortest distance between two points on a plane is a straight line between them. Because of the shortest distance property, the great circle arc between two points on a sphere can be found practically by stretching a piece of string tightly between the two points.

In Figure 1 (left and right) the two great circle arcs joining A and B are AB and $ADCB$. The shorter of these arcs, namely AB, is the shortest distance (on the sphere) between A and B. The absolutely shortest distance between A and B is, of course, the straight line AB which is a tunnel through the sphere. For the antipodes A and C, any arc joining them along any great circle containing them is the shortest distance on the sphere between them. The absolutely shortest distance between them is, of course, the diameter AC through the sphere.

At any point on or inside the earth, *vertical* means the direction of the straight line joining the center of the earth and that point. Thus if Figure 1 represents the earth, then the vertical directions at points A and B are, respectively, OAT and OBU. The great circle arc AB lies *vertically above* the straight line (tunnel) AB, since the points on the former are vertically above the points on the latter.

[8] Sometimes, *azimuth* is measured from south, not north.

An important fact is that *if an object positioned vertically above one location can be seen from another location, that object appears in the direction of the great circle between the two locations*. Thus, tall structures, mountain peaks, and geostationary satellites appear in the great circle direction toward their base point on the earth. To see this, suppose A and B are two points on the earth, and suppose a point T on a tower built at A is visible from B. (Again, refer to the right-hand picture in Figure 1.) Since T is vertically above A, by definition the line TA passes through the earth's center O. Consider the plane defined by the three points T, A, and B. As this plane passes through T and A, it contains every point on TA, and hence it contains the earth's center O. It is, therefore, the plane of the great circle through A and B. Thus, the line BT in this plane lies vertically above the great circle arc BA. Thus the line of sight BT from B to the tower at A is precisely in the direction of the great circle from B to A.

When a heavenly body, such as the sun, the moon or a star, happens to be at a location's zenith (that is, vertically above the location), then the body can be thought of as being at the top of an imaginary, very high tower at that location. From the property mentioned above, it follows that if a heavenly body is at the zenith of some location on earth and is also visible from another location, then it appears there in the great circle direction to the first location.

Elementary geometric reasoning also shows that *objects moving in straight lines above the earth appear to move along great circles*. For example, laser beams and microwave signals travel in great circle directions. These things move in a straight line in space. Their movement on the earth appears to be in the great circle direction between the source and the destination, or the base points of the source or the destination when either or both are elevated points. Essentially, the direction of travel is the great circle cut on the earth by the plane containing the center of the earth and the straight line path in space.

Moving objects that are constrained only by gravity move on the earth along great circles. For example, an object, which is thrown with a great force, and is then let continue under its own momentum, keeps a great circle course, starting with the initial direction of launch. Thus, cannon balls, torpedo boats, and missiles move along great circle directions. Without gravity, their trajectory would be a straight line, vertically above the great circle in the direction of launch. The plane of motion, which contains this great circle, is vertical. Gravity adds only a vertical component to the motion, so the plane of motion remains unchanged and the trajectory has the direction of a great circle.

The bearing generally keeps changing along a great circle on the earth. For example, a great circle path may start out heading northeast, gradually change heading to east, then gradually change it further to southeast. The only great circles with a constant bearing are meridians and the equator. The reason a great circle's bearing keeps changing is that the great circle is as "straight" as is possible on a sphere, but the meridians keep curving underneath it. The fact that a great circle does not make a constant angle with the meridians does not mean that an object on a great circle course has to keep turning. On the contrary, as we have seen, an object moving on the earth's surface will follow a great circle unless external forces cause it to change course.

3.6 Rhumb lines

A *rhumb line*, also called *loxodrome* or *spherical helix*, is a path on the earth's surface which cuts all meridians at a constant angle. The rhumb line's property of always having the same bearing is important in navigation. Ships and airplanes have to be constantly propelled and steered, as they do not have large inertia to continue moving in any one direction. In navigation, one relies on compasses or other directional instruments. It is convenient to maintain the same bearing for long stretches, by manipulating the steering control such that the compass shows a constant angle with north.

On a flat earth, the north-south lines would be parallel, and a line which makes a constant angle with those would be a straight line. On the round earth, the north-south lines are the meridians which are not parallel, and, in fact, converge at the poles. Clearly, a line which makes the same angle with meridians cannot

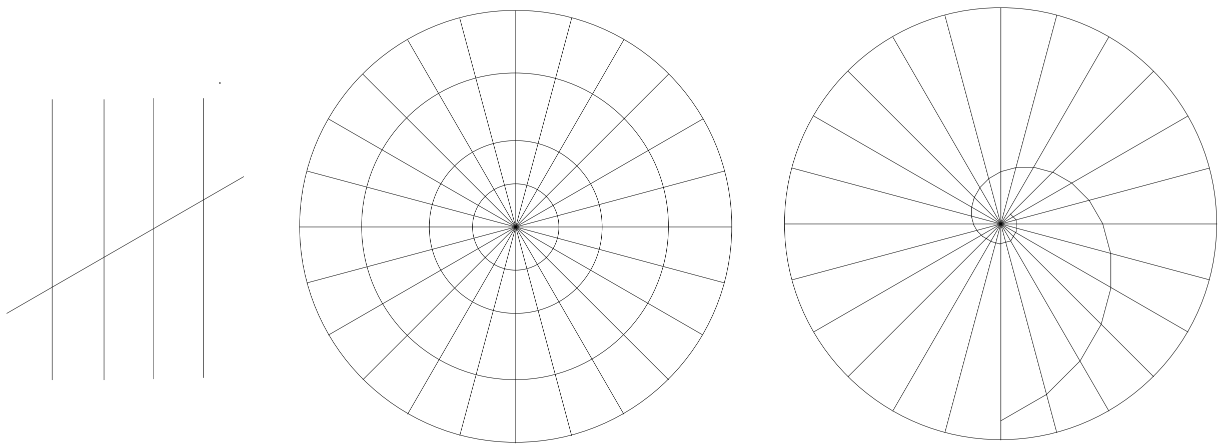

Figure 5: Rhumb lines with bearings of $60°$ on a flat earth (left), $0°$ and $90°$ (center), and $60°$ (right).

be straight. An object moving along a rhumb line, therefore, does not move in a straight course. In fact, such an object keeps subtly turning from a straight course all the time in trying to maintain a constant angle with meridians. Conversely, an object moving unsteered under its own momentum moves on the earth's surface in a great circle trajectory, not a rhumb line. The object cannot move in a rhumb line since the meridians keep curving underneath it, and therefore its angle with them keeps changing.

Figure 5 (left) shows a rhumb line with the bearing of $60°$ on a hypothetically flat earth. The north-south lines are parallel in this case, so we can represent them by vertical lines. The rhumb line is the slanting line that makes a $60°$ angle with each of the north-south lines. In this hypothetical case, the rhumb line turns out to be a straight line.

Figure 5 (center and right) depict the situation on the actual, round earth, and show two sets of rhumb lines in the northern hemisphere. In each of these two pictures, the view is from a point directly above the north pole. The point at the center of the picture is the pole, with meridians emanating from it. In the center picture, the circles are the rhumb lines which maintain a constant $90°/270°$ bearing, that is, the paths headed always east/west. Clearly these rhumb lines are the same as parallels (lines of latitudes) and are therefore circles just as they appear in the picture. The meridians (the radial lines in the picture) are themselves rhumb lines, with $0°/180°$ bearing, that is, headed north/south. Being meridians, these are semicircles but appear as straight lines in this picture.

The right picture shows a rhumb line with $60°$ bearing. This rhumb line is not straight or circular, but *helical* like the thread of a screw. It approaches the pole by spiraling toward it. The path never really reaches the pole although comes arbitrarily close to it. All rhumb lines that are not headed either north-south or east-west are similar, spiral paths on the earth.

Unlike a great circle, a rhumb line that is not a meridian is not contained in a plane. It was shown in Section 3.5 that elevated objects appear in the great circle direction to them. Although the great circle is not a straight line, it lies in a plane, so light reaches from the object to the observer by a straight line path in the plane of the great circle. Generally, as a rhumb line cannot be within a plane, light cannot travel along or vertically above the rhumb line, and therefore it is not possible for an object to appear in the rhumb line direction.

The rhumb line direction between two points on the earth can be determined graphically from a Mercator projection map that will be described later. This is the only projection known in which rhumb lines are transformed into straight lines. On this map parallels and meridians are portrayed as horizontal and vertical

lines, respectively. Thus to find the rhumb line direction between two points, we can mark the two points on the map, join them by a straight line, and measure off the angle of this line from an intersecting vertical line.

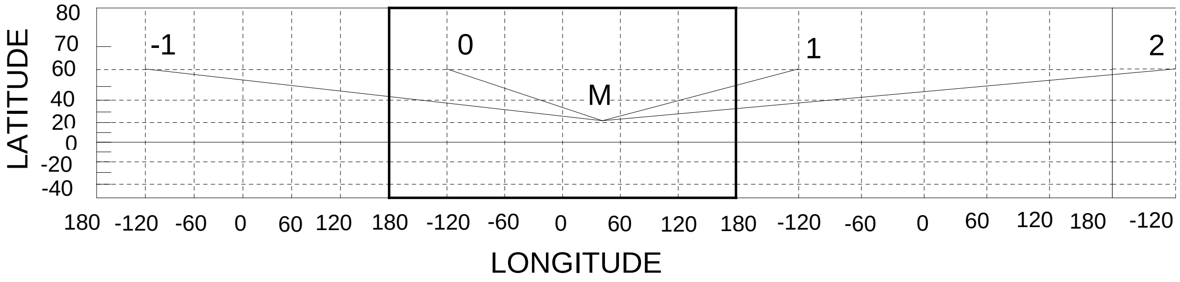

Figure 6: Many rhumb lines for the same two end points.

In Figure 6, the part enclosed by thick lines shows a conventional Mercator projection map. To reduce clutter, the map shows only the latitude-longitude grid, not any countries. The point M represents Mecca ($21°25'$ N, $39°49'$E). The point 0 represents Cameron Hills at the northeastern corner of British Columbia, Canada, with very easily plottable coordinates $60°$ N, $120°$W. The rhumb line direction to Mecca is the bearing of $0M$ which should measure to $108°31'$ E of N.

Rhumb lines are not defined to be unique, and we can generally find many different paths of constant bearing between two given points. Therefore we are usually interested in *the shortest line of constant bearing* between given points. If two points have the same latitude, then between them we have exactly two rhumb lines, namely, the lines with bearings of $90°$ and $270°$ (the lines headed east and west, respectively). However, between two points with differing latitudes, there are an infinite number of rhumb lines. Because the earth is round, a map can be made by pasting end to end any number of copies of the standard Mercator map with $360°$ longitude expanse. A location can be represented by a point on any of these copies. In Figure 6, the standard map is enclosed by thick lines, and an extended map is enclosed by thin lines. Cameron Hills is now represented by points -1, 0, 1, and 2. The corresponding rhumb lines to Mecca have bearings $95°53'$ E of N, $108°31'$ E of N, $104°57'$ W of N, and $95°27'$ W of N.[9] A traveler can reach Mecca from Cameron Hills by maintaining any of these bearings—and an infinite number of others obtained by further extension of the map. The shortest rhumb line is the steepest line, that is, the line with the largest numerical value of bearing, in this case, $108°31'$ E of N.

Now consider Anchorage, Alaska, whose coordinates are $61°13'$ N, $149°53'$W. If we plot the points -1, 0, 1, and 2 to represent Anchorage, we will find the rhumb line bearings to Mecca to be $95°49'$ E of N, $106°26'$ E of N, $108°11'$ W of N, and $96°01'$ W of N. The steepest angle, $108°11'$ W of N, is for the W of N direction, indicating that in the rhumb line sense, Mecca is to the *southwest* of Anchorage. Several correspondents of N&K have claimed that Mecca is to the southeast of North America without bothering to point that for most of Alaska this is not the case.

[9]Instead of determining a rhumb line bearing by measuring it on a Mercator projection map, we can more conveniently—and more accurately—compute the angle by using the formula

$$\tan^{-1}\frac{\lambda_2-\lambda_1+360k}{\ln\frac{\tan(\pi/4+\phi_2/2)}{\tan(\pi/4+\phi_1/2)}}$$

where ϕ_1 and λ_1 are, respectively, the latitude and longitude of origin, ϕ_2 and λ_2 are, respectively, the latitude and longitude of destination, and k is an integer, positive, negative or zero. All angles are in degrees. For the shortest rhumb line, the value of k is one of -1, 0, or 1. The formula can be derived easily from the equations of Mercator projection, given, for example, in [23].

4 Definition of the Qibla Direction

The qibla is the direction to the Ka‘ba, but what precisely is that direction? Two of the possibilities are the great circle and the rhumb line directions from the given place to Mecca. From the description of these directions given in Section 3, and from the commonly understood meaning of “facing” the Ka‘ba (or the Sacred Mosque), it seems natural to adopt the great circle direction for the qibla. For someone who can see the Ka‘ba, the direction sought is, of course, the line of sight to the Ka‘ba. For someone who is farther away but within the visibility range of Mecca’s landmarks or hills, the qibla is the line of sight to these. The definition of qibla is the common sense extension of the idea of line of sight to the Ka‘ba, namely, the line of sight to a vertical line passing through the Ka‘ba. This is the direction, for example, in which an imaginary tower built over the Ka‘ba would appear. It seems indisputable that for someone who could see a tower built over the Ka‘ba, the qibla would be the line of sight to the tower. A classical definition in this spirit is given by Ibn al-Haitham as follows (translated from the Arabic text quoted in [15, p. 11]:

> The qibla is the direction such that when a human observer faces it, it is as if he is looking at the diameter of the earth passing through the Ka‘ba.

From the properties of great circles discussed in Section 3.5, we know that this line of sight is precisely in the great circle direction to the Ka‘ba. So Ibn al-Haitham continues:

> And the ray coming out of his eye in that direction is in the plane of the great circle passing in the direction of his zenith and the point corresponding to [the zenith of] Mecca.

Here Ibn al-Haitham mentions the great circle on the celestial sphere passing through the zeniths of the location and Mecca, instead of the terrestrial great circle through the two places. Muslim astronomers often transformed the qibla determination problem into some familiar problem of astronomy by projecting the points on the earth to those on the celestial sphere.

Ibn al-Haitham’s simple and precise definition is described in several alternative ways in the jurisprudential literature (see, e.g., [5] for this discussion). For example, one is supposed to be facing the qibla when a straight line from one’s chest or forehead reaches “the air above the Ka‘ba.” (Here, the chest or the forehead is considered an arc, and the line is meant to be a radial line passing through the middle point of the arc.) The literature uses a variety of geometrical configurations to discuss the concept of qibla orientation and the allowable approximations, since exact orientation is not required according to most jurisprudents. Jurisprudential considerations are beyond our scope. The purpose of mentioning them is that they all contain a common theme that the qibla is along the line of sight to a vertical point above the Ka‘ba. This can only be the great circle direction, not the rhumb line which is not the line of sight as we have seen in Section 3.6. The rhumb line’s property that it makes a constant angle with the meridians is irrelevant to the qibla. Turning towards the Ka‘ba is tantamount to making an attempt to see it, not to navigate towards it with an artificially constrained movement. Moreover, as we have seen, the rhumb line direction is not unique.

Section 3 has shown that the direction of the straight line in space between two points on the earth, that is, the direction of the straight tunnel between the two points, is also the great circle direction between the two points. The line of sight idea cannot be used everywhere on earth, but the idea of the tunnel can. This is because even an infinitely high tower built in Mecca would be visible only from one half of the earth. To see this, first consider the problem of determining the direction of the north pole from any given location on the earth. If a tower built on the north pole can be seen from a location, then the line of sight is the north direction. We can think of the star Polaris as being at the top of a very high imaginary tower at the North pole.[10] So the direction in which Polaris is seen is the (great circle) direction to the north pole. But due to

[10] This is only approximate because Polaris is off the true north by slightly less than one degree.

the earth's curvature, Polaris appears lower and lower in the sky as we move away from the north pole, until it appears on the horizontal level on the equator, and then disappears altogether in the southern hemisphere.

In the same way, a tower built in Mecca, no matter how high, will be visible from only half of the earth, because in the other half the tower will be hidden by the earth's curvature. What corresponds to the equator in this case is the great circle that lies midway between Mecca and its antipode. This circle, which joins all the points that are at 90° distance from Mecca, divides the earth into two hemispheres such that even an infinitely high tower in Mecca can be seen only from the hemisphere which contains Mecca. Of course, if the tower is imagined to be infinite in both directions (Ibn al-Haitham's "diameter" through Mecca), then it can be seen from anywhere on the earth. Although still in the direction of the line of sight to this tower, the qibla will be in the opposite sense, if the place of observation and Mecca are in different hemispheres.

There is no difficulty in defining the qibla by the great circle direction. In the hemisphere of Mecca, this is also the line of sight to some point vertically above Mecca. But in both hemispheres, the great circle direction is the most direct path to Mecca, being the direction of the straight tunnel to Mecca.

5 Qibla Determination Methods

A passage (in free translation) from al-Razi's *Tafsir al-Kabir* [20, vol. 4, p. 131] beautifully motivates why in determining the qibla one sometimes needs to *calculate*, an act much disparaged by some theologians:

> The qibla direction is the intersection point between the circle of horizon and the great circle passing in the direction of our zenith and Mecca's zenith. The qibla angle is the arc on the circle of horizon between the qibla direction and the meridian of our city; and [the angle] between the qibla direction and the equinoctial sunset point [i.e., west][11] is the complement of the qibla angle. They say: And in finding the qibla direction, one needs to know the longitude and latitude of Mecca. If the longitude of the city equals the longitude of Mecca, and its latitude differs from the latitude of Mecca, then its qibla direction is along the meridian, to the south if the city is to the north [of Mecca], and to the north if the city is to the south. If the latitude of the city equals the latitude of Mecca, and its longitude differs from the longitude of Mecca, then it may be conjectured that its [i.e., the city's] qibla direction is the east-west line; however, this is a wrong conjecture.[12] And [in fact] it is also possible for some cities whose longitudes and latitudes differ from Mecca's longitude and latitude that their qibla direction be the equinoctial sunrise or sunset point [i.e., east or west]. If this is the situation, then there is no choice but to derive [i.e., calculate] the qibla angle.

As al-Razi mentions, knowing just one's relative east/west and north/south position with respect to Mecca is not enough to correctly infer the qibla. So in this section, we resort to some high-school level mathematics to describe how to compute the qibla.

5.1 Basic Spherical Trigonometric Formula

The problem of qibla determination has a simple formulation in spherical trigonometry. In Figure 7, A is a given location, K is the Ka'ba, and N is the North Pole. The great circle arcs AN and KN are along the meridians through A and K, respectively, and both point to the north. The qibla is along the great circle arc AK. The spherical angle $q = NAK$ is the angle at A from the north direction AN to the direction AK

[11] Only on the days of spring and fall equinox, around March 21 and September 21, does the sun rise exactly from the east and set exactly in the west. On other dates, the sunrise and sunset directions deviate from east and west. The deviation, which is more pronounced at higher latitudes, is the largest on the days of summer and winter solstice, around June 21 and December 21. Al-Razi is characteristically meticulous in specifying the east and west as the directions of sunrise and sunset *during equinoxes.*

[12] The Mercator map reinforces this common fallacy.

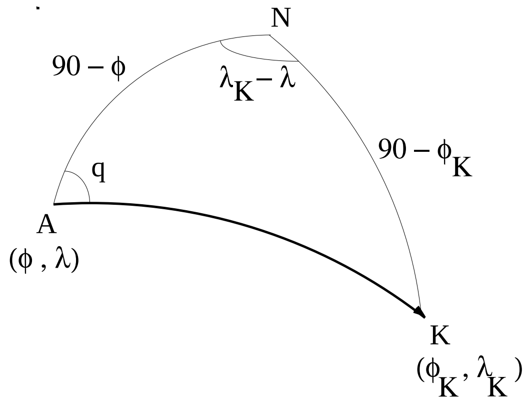

Figure 7: Spherical triangle for determining the qibla

towards the Ka‘ba, and so q is the qibla bearing to be computed. Let ϕ and λ be the latitude and longitude of A, and ϕ_K and λ_K be the latitude and longitude of K (the Ka‘ba). If all angles and arc lengths are measured in degrees, then, by comparing with Figure 4, it is seen that the arcs AN and KN are of measure $90 - \phi$ and $90 - \phi_K$, respectively. Also, the angle ANK between the meridians of K and A equals the difference between the longitudes of A and K, that is, $\lambda_K - \lambda$, no matter what the prime meridian is. Here we are given two sides and the included angle of a spherical triangle, and it is required to determine one other angle. One of the simplest solutions is given by the formula

$$q = \tan^{-1} \frac{\sin(\lambda_K - \lambda)}{\cos\phi \tan\phi_K - \sin\phi \cos(\lambda_K - \lambda)}. \qquad (1)$$

Mathematical handbooks and textbooks on spherical trigonometry and spherical astronomy often give a number of alternative formulas or sequence of formulas equivalent to the above equation. Several of these formulas are collected in Bagvi[3, 4], including the “half-angle”, Napierian, and “haversine” formulas. These formulas were very useful when computations were done by hand using trigonometric and logarithmic tables, and side results were often needed to cross-check the computation. Nowadays there is no advantage in any of these formulas over Equation (1).[13]

In Equation (1), the sign of the input quantities are assumed as follows: latitudes are positive if north, negative if south; longitudes are positive if east, negative if west.[14] The quadrant of q is assumed to be so selected that $\sin q$ and $\cos q$ have the same sign as the numerator and denominator of Equation (1). With these conventions, q will be positive for bearings east of north, negative for bearings west of north.[15]

Example

To find the qibla for Washington, D.C. and Anchorage, Alaska.

According to an official survey, the geographical coordinates of the Ka‘ba are: $\phi_K = 21°25'24''$N $= +21°.423333$ and $\lambda_K = 39°49'24''$E $= +39°.823333$. For Washington, D.C., we use the coordinates: $\phi = 38°54'$N $= +38°.9$, $\lambda = 77°01'$W $= -77°.016667$. Substituting these values in (1), we obtain $q = +56°.575960 = 56°35'$ E of N.

[13]The qibla computation literature often interchanges the numerator and denominator of this equation to make it a “cotangent formula.” The `arctan` form is more convenient since many calculators do not provide the `arccot` function directly.

[14]The formula on p. 289 of [1] is slightly different because there I used the opposite sign convention for longitudes.

[15]Software libraries of mathematical functions usually have a two-argument function `atan2`. If the numerator and the denominator of the above equation are used as arguments of this function, with the signs of coordinates chosen according to the above convention, then the resulting qibla angle automatically has the absolute value between 0 and 180 degrees, and its sign is exactly according to the above convention.

For Anchorage, Alaska, we use the coordinates: $\phi = 61°13'\text{N} = +61°.216667$, $\lambda = 149°53'\text{W} = -149°.88333$. Substituting these values in (1), we obtain $q = -9°.098363 = 9°06'$ W of N.

5.2 Direct Solar Observation

When the sun happens to be exactly vertically above Mecca, it can be thought of as being at the top of an imaginary, very high tower built in Mecca. Wherever on the earth the sun is visible at that moment, the direction in which the sun appears is the exact direction of the qibla. So a favorite method used by classical Muslim scientists for determining the qibla was to observe the sun at that moment.

Here is a description[24, Vol. I, p. 306] of this method by al-Tusi (1201–1274 A.D.):

> The sun transits the zenith of Mecca when it is in degree 8 of Gemini and in [degree] 23 of Cancer at noontime there. The difference between its noon and the noon of other localities is measured by the difference between the two longitudes. Let this [latter] difference be taken and let an hour be assumed for each 15 degrees and 4 minutes for each degree. The resulting total is the interval in hours from noon [for that locality]. Let an observation be made on that day at that time—before noon if Mecca is to the east or after it is to the west; the direction of the shadow at that time is the qibla bearing.

This terse description requires some explanation. Twice a year—on approximately May 28 and July 16—the sun happens to pass exactly above Mecca at noontime.[16] Al-Tusi specifies these two days by the sun's position on the ecliptic. He describes how to calculate the observation time at any location from the longitude difference between Mecca and the place of observation. Of course, nowadays if there were to be a radio broadcast from Mecca at noon on those two days, then one would be able to determine the qibla just by observing the shadow at that time, without having to know the latitude or longitude of Mecca or of one's own location, and without having to do any computation.

Although elegant, this method is not very useful, specially for North America. The precise determination of the qibla is possible only on two days in the entire year, and for a very brief moment on each day.[17] Moreover, as we have seen in Section 4, this method works only in a half of the world. The great circle lying midway between Mecca and its antipode divides the earth into two hemispheres. If, at the time when the sun is exactly above Mecca, one were to make an observation on the locations on the dividing circle itself, the sun would appear on the horizontal level. On the other side of this circle, the sun would not be seen at all at that time.

This dividing circle cuts through North America in a northwestern direction, passing near Boston and Montreal, and the method is usable only at points east of that circle. This excludes most of North America. The method works in all of Maine and New Brunswick; it barely works in Boston and Montreal; and it does not work at all in New York and Ottawa. This can be verified easily on a globe, using a piece of string to measure angular distances, since the method can only work in those places whose angular distance from Mecca is less than $90°$. Another simple way to see this is to check whether on May 28 the time of noon in Mecca happens during day time in the city of interest; otherwise, of course, the sun cannot be seen at the required moment. Normalizing all times to Mecca, the noon in Mecca is at 12:18, while the sunrise in Boston, Montreal, New York and Ottawa are at 12:12, 12:12, 12:29, and 12:20, respectively. Note that the observation times are before sunrise in New York and Ottawa.

A method similar to the above can be used at night time by observing a star at the moment of its zenith transit over Mecca. But this requires some expertise of astronomy, and is again not practical for most of

[16] When the sun's declination is the same as the latitude of some location, the sun's position at noon there is at the zenith, that is, vertically above that location. It can be checked by computation or by consulting a table such as [1, p. 288] that the sun's declination on May 28 and July 16 is $+21.4°$, about the same as the latitude of Mecca.

[17] The method can also be used on dates close to May 28 and July 16 for determining the qibla *approximately*.

North America.

5.3 Shadow Method

After the qibla bearing has been computed, one still needs to determine the (true) north direction, and then find the qibla by measuring the computed angle from north. Both these operations are subject to measurement errors. A method that bypasses these operations altogether is to observe the shadow of a vertical object at a time when the shadow makes an easily measurable angle such as $0°$, $90°$, $180°$, or $270°$ with the direction of the qibla. The angles $0°$ and $180°$ imply that the qibla is exactly in the same direction or exactly opposite to the shadow, so at these times there is no angle to measure and the qibla can be found very accurately. The other two angles imply that the qibla is perpendicular to the shadow, so again the qibla can be found quite accurately. Computing such times is a simple problem of astronomy. A description of this method can be found, for example, in [1, pp. 290–292].[18]

5.4 Other Methods

Twelve centuries of research have produced such a vast array of qibla determination methods that they can be covered only in large, dedicated volumes. There are both approximate and exact methods, and the techniques they utilize include graphical constructions, mathematical (trigonometric) computations, astronomical observations, use of shadows, and specialized "qibla indicator" instruments. Most of these have little practical utility nowadays, since equations such as (1) of the previous section provide accurate answers, and are relatively easier and more efficient to use. On the other hand, these methods often show great ingenuity, are fascinating mathematical reading, and are of much historical importance. The interested reader may explore the literature, starting with the references in Sections 2.2 and 2.3.

Much work has also been devoted to compiling qibla tables, which list the qibla either for specific cities or as a function of latitudes and longitudes. Classical tables use the values of longitudes which are based on prime meridians that are now obsolete. Among modern tables, I have already mentioned Baghayiri[2] and Husayn[12] as representative work. The former gives the qibla for most of the well-known cities of the world. The latter tabulates the qibla for the range of latitudes from $89°$ N to $89°$ S and longitudes from $179°$ E to $180°$ W for each degree of latitude and longitude. I have also seen works that provide similar data in the form of graphs rather than tables.

There is scarcely any need for producing more of these once-admirable works of reference, since any interested person can now generate them instantly on a computer. Qibla tables, much like trigonometric tables, are much less needed now since it is nowadays easier to generate required values on demand than to look them up in a table.

6 Qibla Maps. Map Projections

Maps are often consulted for finding the qibla. Depending on how a map has been constructed, it may or may not portray the qibla accurately. So in this section, I discuss several kinds of maps and their suitability for the purpose of determining the qibla.

The best model of the earth is a globe. Only a globe can faithfully depict shapes, areas, distances, angles, and directions as they actually occur on the earth. In particular, a globe shows the qibla most accurately. But globes are quite limited in scale of representation, and are not as convenient to produce, duplicate or

[18] The author's program *Minaret* for prayer schedule and calendar computations also provides qibla determination times based on this method. This program, which runs on PCs and Macintosh computers, is available for anonymous ftp from the same directory as this paper.

disseminate as maps made on sheets of paper. So flat maps remain popular in spite of their distortions and relatively inaccurate portrayal of geometric relations on the earth.

A *map projection* is a method of representing the points on a sphere by points on a plane. A map projection is usually given in the form of mathematical equations that relate the geographical coordinates (latitude and longitude) of each location on earth to the familiar (x, y) coordinates of the point on a plane map which represents that location. A plane map can at best correctly show only a few of the geometric relations on the sphere, so different map projections are needed for faithful portrayal of different combinations of relations of interest. The subject of map projections is practically important and mathematically fascinating, so hundreds of map projections have been devised, and new ones are continually being invented. A detailed discussion of map projections is beyond our scope. I'll mention only a few projections that are relevant to the topic. An encyclopedic description of map projections is [22], and maps made with most of the known projections can be found in [23].

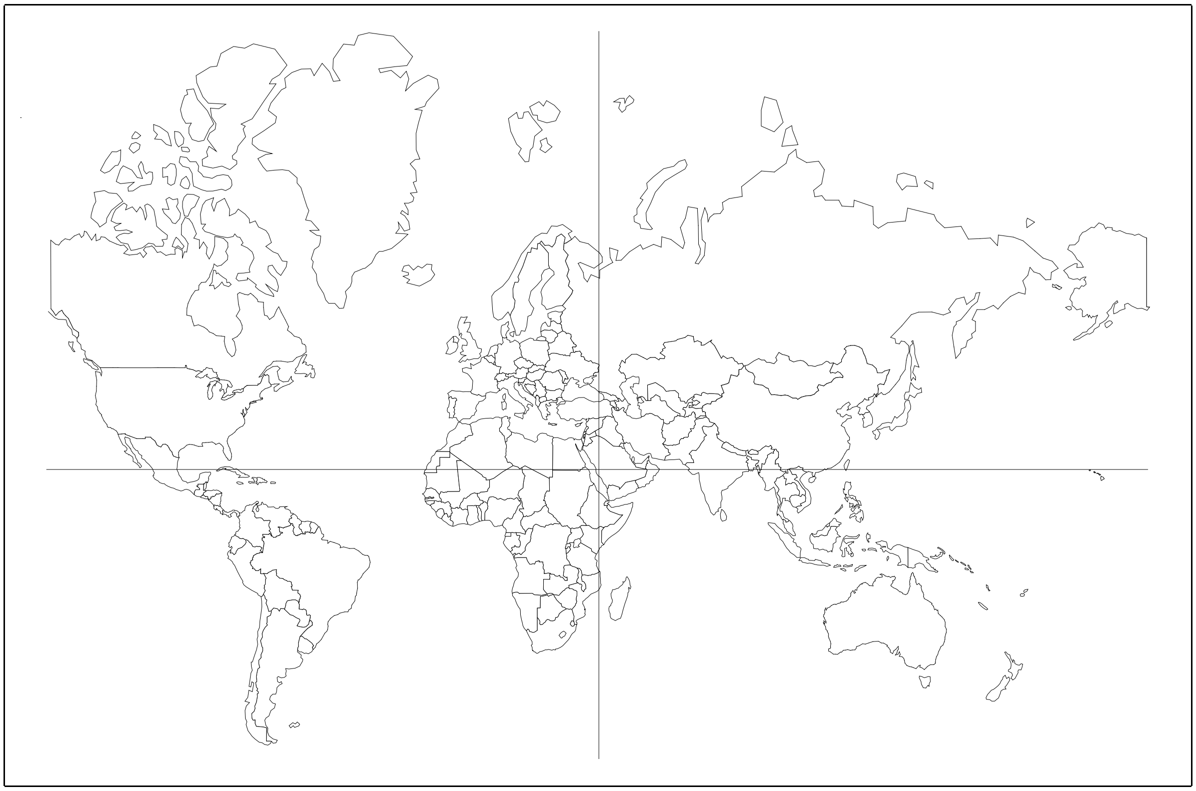

Figure 8: Mercator projection world map centered on the meridian of Mecca.

The most familiar map projection is the *Mercator projection.* Unfortunately, this projection is also the most misleading when it comes to determining the qibla, so I describe its construction in some detail and point out its disadvantages.

A Mercator projection map is given in Figure 8. In this projection, parallels and meridians are represented by horizontal and vertical lines, respectively. As we have seen parallels decrease in length from about 40,000 kilometers at the equator to zero at the poles. On a Mercator map, the lines representing the parallels are stretched out to the same length at every latitude. Thus, the horizontal scale continuously increases with higher latitude. To compensate for the distortion of shapes due to this horizontal spreading out toward the pole, the vertical scale is also stretched in the same proportion. The poles and locations close to them cannot

be represented on a Mercator map since, due to the scale stretch, the poles project to infinity and the points near the poles project so far away that the map would have to be impractically large. Since scales stretch both horizontally and vertically, their multiplicative effect distorts areas grossly. For example, on a Mercator map Africa and Greenland appear to be of the same size while in truth the area of the former is nearly 14 times larger than that of the latter.

The unique and important feature of the Mercator projection map is that rhumb lines are represented by straight lines. A navigator, who is trying to determine a rhumb line course between two locations, can simply draw a straight line on the map between the two points, then measure the angle between this line and the vertical line to get the bearing of the course. Actually, these days it is simpler and more accurate to compute the rhumb line direction by a trigonometric formula than to measure the angle on a map. (Such a formula is given in a footnote on page 14.) Nevertheless, it is convenient to be able to see the rhumb line directions throughout the world at one glance.

As we have previously seen, parallels or the small circles of latitude are east-west directions only locally. Given a point which is not on the equator, the points east/west of it lie not along the small circle but along a great circle. This great circle is tangent at the given point to the parallel (small circle) passing through that point. The Mercator map is deceptive in depicting parallels as true east-west lines.

We have also seen that the correct qibla direction is not the rhumb line but the great circle. Great circles appear as curved paths on the Mercator projection, the curvature being more pronounced for longer paths. Hence except for short distances, the Mercator maps are not suitable for inferring the qibla.

Another important map projection is *azimuthal*. The construction of an azimuthal map is best understood by the following imaginary experiment. Take a transparent globe, such as one built with clear plastic (such globes are actually available commercially these days), and place it near a projection screen. On the line which is perpendicular to the screen and passes through the globe's center, choose a point somewhere on the side of the globe opposite to the screen. Then on this point place a tiny light source. As a result, the map painted on the globe will project on the screen. To get a less cluttered map, a part of the globe can be sawed off so that only a portion of the globe's surface gets projected. The picture on the screen, whether of a part of the globe or the whole globe, will be an azimuthal projection map.

Depending upon the relative placement of the screen, the globe, and the light source, very differently looking maps are obtained, but they are all azimuthal. Technically, therefore, azimuthal is not just one projection but a whole family of projections. The physical operations mentioned above are for explaining the principle of azimuthal projection only; actual maps are plotted quite conveniently by using the mathematical equations describing the projection.

The azimuthal projections have the following properties:

1. All great circles through the center of the map appear as straight lines on the map.

2. All points equidistant (on the earth) from the center of the map appear equidistant on the map also.

3. The stretching of scale and distortion of shapes increase with distance from the center of the map.

An azimuthal map centered on the north pole is given in Figure 9. Such a map is obtained by placing the light source at the south pole of the globe and the projection screen touching the globe at the north pole. This map is called a *polar azimuthal equidistant projection map* or simply a *polar map*.

This map correctly shows that the land masses of Asia, Europe and North America are tightly grouped together near the north pole. By contrast, the Mercator projection (Figure 8) gives the illusion that they are far apart and isolated. For example, Siberia and the Canadian Northern Territories are really rather close, just across the north pole from each other—not half the world apart as the Mercator projection shows.

Property 1 above implies that the azimuthal map of Figure 9 shows the great circles passing through the north pole as straight lines. These are, indeed, the meridians, that is, the north-south lines, appearing as

Figure 9: Azimuthal projection map centered at the North pole.

radial lines on the map. The circles are the parallels, that is, the east-west lines. A direction at any point can be read off on the map by noting the angle the direction makes with the meridian at that point.

While on this map only the directions from the north pole are displayed exactly, the straight lines passing close to the north pole represent great circle directions approximately. In fact, as long as a straight line is within the northern hemisphere, it is quite a reasonable approximation to the great circle direction. This map is very useful in air navigation and telecommunication where great circle routes are important. For example, by drawing the corresponding straight lines on this map, we find that the great circle direction from Miami to Kuala Lumpur is almost exactly north, from Los Angeles to Tokyo is northwest (nearly $45°$ W of N), and from Chicago to Rome is northeast (nearly $45°$ E of N). This map also gives a fairly good approximation to the direction of qibla from all of North America. For example, a straight line from Houston to Mecca on this map is close to northeast ($45°$ E of N), and so gives the qibla almost exactly.

From Property 1 mentioned above, it follows that if we construct an azimuthal map in which Mecca is projected at the center, then all great circles passing through Mecca will project as straight lines. In other words, the direction of the qibla from any location in the world can be found on this map by drawing a straight line from that location to Mecca. If we also plot the meridians on the map, then the bearing of the qibla at any location can be read off as the angle between the meridian at that location and the line from that location to Mecca. Such a map is given in Figure 10. Although it may have the appearance of a “picture” of the globe, it really is a plane map. Note that while in a picture one can see at most one half of the globe,

Figure 10 shows the entire surface of the earth.

The azimuthal equidistant projection, so convenient for displaying the qibla, was first invented by none other than al-Biruni for constructing plane maps of the celestial sphere. (see Berggren[7]). The scale and shape distortion in this projection increase with the distance from the central point. For example, Figure 10 shows substantially distorted shapes of North and South America, which are more distant from Mecca than other continents. Nevertheless, the shapes are quite recognizable.

There are many other projections (see, for example, [22, 23]) that transform certain families of great circle directions into straight lines, and therefore may be pertinent to the construction of "qibla maps." One such projection, which also belongs to the family of azimuthal projections, is called *gnomonic*. Referring again to our imaginary experiment, the gnomonic projection results when the light source happens to be at the center of the globe and the projection screen just touches the globe. The map is unique in possessing the property that *all* great circles project as straight lines. Such a map is quite useful for navigation and communication where great circles play an important role. The direction of the most direct air travel route between any two points on the map can be determined by simply connecting the two points with a straight line. However, gnomonic projection maps suffer from some serious drawbacks: A finite-sized map can only represent a part of a hemisphere, not the whole world, and the scale and shape distortions severely affect this map. In practice, only a small region of the world can be drawn, since gross shape distortions occur as the depicted area increases.

In some projections, great circles from any point to *two special points* project as straight lines. In yet another (the *Littrow Projection*), great circles from any point to *any point on a special meridian* project as straight line. There is, moreover, a *Mecca projection*, also called *Craig Retroazimuthal projection*, in which all great circles to a special point project as straight lines, and all meridians project as *vertical* straight lines. The idea is that, because of familiarity with maps such as Mercator, people can more easily visualize directions when the north direction is shown as always vertical and up. Craig, an Englishman who worked in the Egyptian Survey Department in the 19th century, designed this projection specifically as the basis of a "qibla map," and described it in a monograph on map projections[9]. In Snyder and Voxland[23] there is a Mecca projection map on p. 150. (But the correct directions are for St. Louis, not Mecca!) This map distorts shapes so much that they become unrecognizable.

What is the best projection for constructing a "qibla map?" Such a map should make it easy to find the great circle direction of Mecca from any place. The gnomonic, Two-Point, Littrow, and Mecca projections attempt to do more than what is really necessary for our purpose, that is, project the great circles passing through Mecca as straight lines. The first three of these produce straight lines for great circles directions to points other than only Mecca. The last one makes all meridians vertical straight lines. These extra accomplishments come at the cost of severe distortions that make these maps rather difficult to read. The azimuthal equidistant projection centered on Mecca does only what is needed, and does it very well. The literature on map projections is vast, and the possibilities of developing new projections are unlimited. Perhaps new projections displaying the qibla in efficient and elegant ways will still be discovered. At present, the Mecca-centered, azimuthal equidistant projection seems to be quite satisfactory. Anyway, qibla maps, like qibla tables, have limited utility now, since the qibla can be very accurately computed on demand for any location instantly and effortlessly.

7 Conclusion

This paper has been motivated by the publication of N&K's book[21] which opposes the choice of the great circle direction for defining the qibla. The book displays a counter-scientific attitude and a primitive understanding of the subject, and yet includes some seemingly scientific arguments. The book also includes religious rulings confirming its views but, with a few exceptions, without taking any scientific stand. If the

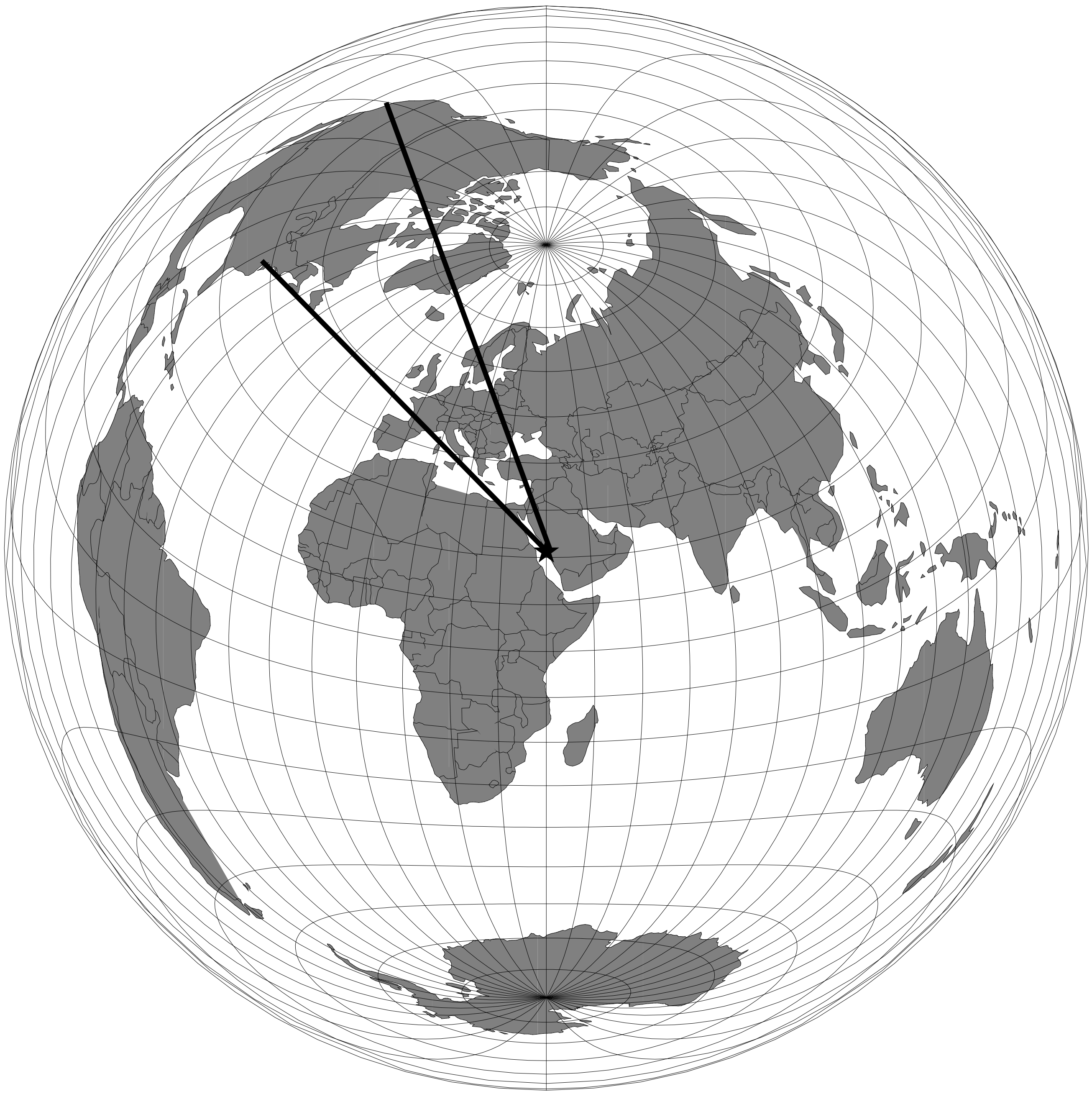

Figure 10: Azimuthal projection map showing the true direction to Mecca from anywhere in the world. (This is not a “picture of the globe” but a plane map.) The arrows show the qibla from Washington and Los Angeles.

book had expressed only the authors' own views and the religious rulings, I would not have reacted to it. But the book also enlists support from several professional scientists, who have argued that the qibla should be in the rhumb line direction. Although this opinion, also wrong, is different from the authors' own, these letters seem to lend some credibility to the book. I therefore felt that a response to the book was warranted.

This paper's main interest is in rectifying the confusion created by, of all things, the statements from scientists. An examination of the book has revealed in it an incredible amount of misconceptions about some elementary geographical and astronomical notions whose correct understanding is essential for the subject of qibla determination. Unfortunately, the confusion is so pervasive that even some of the scientific professionals have not escaped it entirely. So this paper has attempted to explain the qibla determination problem starting from very basic concepts, and has tried to clarify several serious misunderstandings that seem to be at the root of the controversies.

The concept of qibla had been thoroughly formulated, and exact mathematical solutions of the qibla determination problem had been found around the tenth century. This was six centuries before the Mercator projection and the rhumb line were introduced. Crude precursors of these did exist, such as the projection of Marinos of Tyre (2nd century A.D.), and the various approximations to the qibla that are somewhat reminiscent of the rhumb line. Scientists like al-Biruni analyzed these pre-Mercator concepts and dismissed them as inappropriate for use in determining the qibla. The introduction of the rhumb line concept never gave any reason to the scientists interested in the qibla problem to have second thoughts about the great circle definition of the qibla.

Thanks to the misunderstanding by some geographers about the rhumb line—for example, that it really is a straight line—there is a controversy in the 1990s whether the qibla should be in the great circle or the rhumb line direction! So this paper has compared in detail the concepts of great circle and rhumb line, and has tried to reason why the former gives the correct direction for defining the qibla. I have also used the opportunity to pack together a lot of elementary but relevant scientific facts that will, hopefully, clarify the subject, and help prevent similar controversies arising from scientific misunderstandings.

8 Acknowledgment

I gratefully acknowledge the assistance I have received from several people. Dr. Omar Afzal provided me some crucial reference material, and made very helpful suggestions. Khalid Shaukat made some very perceptive comments on a draft. My brother Masood went through much trouble and expense in obtaining a rare copy of [5] for me. Finally, Dr. Muzzammil Siddiqui brought N&K's book to my attention in the first place.

A Appendix: Additional Comments

This section discusses some comments made in [21] by the authors and their correspondents. I've tried to make this section independent, so there is some repetition of the material elsewhere in the paper. Unless stated otherwise, page numbers refer to [21].

A.1

On p. 66, there is a letter from Dr. Klinkenberg of the University of British Columbia with the following text:

> ...Having read over the letters presented to me (from McGill, Concordia, the National Geographic Society, Carleton, the National Research Council of Canada, and McMaster), and reviewing my previous correspondence, I am even more confident that the true direction one must

> face from North America to Mecca is the bearing, which for North America is East Southeast. If a tower were to be built in Mecca that could be viewed from North America, that tower would appear on the East Southeast horizon.

Identifying the qibla of a location with the direction in which a tower in Mecca would appear from that location is an excellent idea. Alas, Dr. Klinkenberg is mistaken that the tower would appear to the east southeast (the rhumb line direction) when viewed from North America. Quite the contrary, the tower would appear exactly in the great circle direction, generally northeast in most of North America. The following elementary geometrical reasoning shows this. Let A be a given location on the earth, M be Mecca, and B be a point on the tower (i.e., vertically above M) which is visible from A. The vertical line BM by definition passes through the center of the earth. Consider the plane defined by the three points M, A, and B. As this plane contains the line BM, it also contains the point at the center of the earth, and is, therefore, the plane of the great circle through A and M. Thus, the line of sight from A to the tower in Mecca, namely, AB, lies vertically above the great circle arc AM, and is in the great circle direction to Mecca.

A.2

The National Geographic Society's Ms. Ryan has this to say (pp. 61–62):

> ...the great circle route [from Washington, D.C. to Mecca] is not a straight line, but rather an arc that begins its ascent going northeast and eventually descends going southeast. By contrast, a straight line from Washington, D.C. to Mecca follows a straight path in an east southeast direction.

This statement is misleading because it fails to mention that it is describing the situation only on one particular representation of the earth's spherical surface on a plane, namely, the Mercator projection map. On the earth's surface itself there is no such thing as a straight path. The great circle is the closest thing to a straight line there can be on a sphere. On the other hand, the east southeast "straight line" she mentions is the rhumb line which is a spiral path on the actual surface of the earth. National Geographic's views seem to have changed since the days of Wellman Chamberlin. (See the *Washington Daily News* story in Section 2.)

A.3

Dr. Frost of Concordia University writes (p. 71):

> ...[In Montreal] observe the location of the sun at noon (12:00) Eastern Standard Time. A vertical stick will cast a shadow that is exactly N-S and the required angle $104°$ E can be easily be applied to that line...
>
> The method outlined above ... relies on the fact that Montreal is located at $75°$ W, the central meridian of Eastern Standard Time.

Noon, the time of sun's meridian transit and the time when shadows point north-south, should not be confused with 12:00. All that Montreal's special location does is to make its standard time equal to its local time. But the local time of noon varies from day to day (a table is given, for example, in [1, p. 288]) and is seldom the same as 12:00. Moreover, suggesting the use of noontime shadows to determine the north direction is bad scientific advice. Noontime shadows are very short, and extending short lines introduces relatively large angular errors.

A.4

The letter (p. 68) from Professor Moore of McGill University includes this:

> Question: What is the direction to face from Montreal to Mecca, such as that Mecca is reached, or the air above it?
>
> Answer: The rhumb line direction from Montreal to Mecca is $104^\circ 30'$ from true north, or $14^\circ 30'$ south of east.

The question, a traditional phrasing of the qibla problem, is about a line in space from a point on the earth to a point vertically above Mecca. The answer is wrong, but perhaps this is so because the question was misunderstood to mean that some one wants to physically "reach" Mecca by travelling toward it. But no such explanation can be given for the following remark (p. 72) from Professor Binghardt of McMaster University:

> QUESTION: If we are standing in Hamilton, in precisely which direction must we face so that our chest will be facing toward the Kaaba (Mecca) or the air above the Kaaba along that straight line?
>
> ANSWER: The straight line between Hamilton, Ontario, and Mecca, which never changes its direction, is at $12^\circ 20'$ (12 1/3°) South of East . . .
>
> On the globe, a great-circle route is the shortest route between Hamilton and Mecca, but this line is constantly changing direction. The line begins at 30° North of true east but ends up running southward into Mecca.

The question could not have been clearer. It is not about the direction of travel but about a line in space from the chest of a person standing in Hamilton to a point above Mecca. The response is misleading by giving the impression that the rhumb line and not the great circle is the direction in which elevated objects are seen on the earth.

A.5

The response displayed on p. 37 from the President of Al-Azhar is a religious ruling. Of concern here is only the scientific part, attributed to Dr. Al-Fandi, which is as follows:

> 1. The mathematical formulas for the triangular spherical computations are all sound.
>
> 2. The application that appeared in your request is wrong, since it was necessary to subtract the resulting figure from 180 degrees in compliance from with the mathematical rules.
>
> 3. The application shown in your request is scientifically wrong and is discarded. Accordingly, the direction of *al-Qiblah* in the city of Montreal is at an angle of 180 degrees minus 58 degrees, which is 122 degrees from due north, and it is southeast.

The calculation sent with the questionnaire has not been given, but there are enough clues in the mention of spherical trigonometry, Montreal, and 58° in the answer. Using the geographical coordinates for Mecca and Montreal given in any correct gazetteer, the spherical trigonometric formula will yield a value of the qibla close to 58–59° east of north. For example, [1, pp. 289, 270] gives:

> Mecca : latitude $= 21^\circ\ 27'$ N, longitude $= 39^\circ\ 45'$ E
> Montreal: latitude $= 45^\circ\ 31'$ N, longitude $= 73^\circ\ 33'$ W
> The qibla for Montreal: $58^\circ 44'$ E of N.

Similar values are also found in other tables, such as [12, 15] for which the computations have been performed independently.

The calculation has been designed to give the angle from (the true) north. If we subtract the value from $180°$, the resulting angle will only be correct if measured from the south. To require this angle supplementation operation just to force the computation to yield a prejudged result for Montreal is an expedient without any foundation.

While the extra operation produces convenient answers for North American locations, its results for other locations will not be pleasing. Let's take Cairo, for example, where the result will be a northeastern direction for the qibla if we follow Dr. Al-Fandi's prescription. With Cairo's coordinates taken as $30°03'$ N, $31°17'$ E, the calculated angle is correctly obtained to be $136°20'$ from north, which is southeast. But by subtracting this from $180°$, we'll get the qibla to be $43°40'$ from north!

A.6

On p. 21, N&K say:

> Furthermore, in their calculations, *ˆAbdullah al-ˆAbdaliyy* and those who followed him relied on that earth is a perfect sphere. Either intentionally or out of ignorance, they did not acknowledge that the shape of the Earth, in truth, is oblate. The oblate shape of Earth was known to the Muslim scholars a long time ago and as was established by the later western astronomers as well.

In [1, p. 289], it is stated: "The direction of qibla at any point on the earth's surface, *assuming the earth to be a perfect sphere*, [italics added] is given by the great circle passing through that point and the Kaaba at Makkah." Since the author states it explicitly *as an assumption*, he obviously does not mean to delude himself or others that the earth is a perfect sphere! But, more importantly, this is a very reasonable assumption for the purpose of determining the qibla. When we want to paint a wall and need to estimate the amount of paint needed, we assume that the wall is a perfect rectangle. For the purpose needed, it is unnecessary to consider the always present minor irregularities which prevent the wall from being a rectangle or, indeed, from being a plane surface in the first place. Moreover, we compute the area in square feet, or in tens of square feet, not in square inches with 100 decimal places. In general, we evaluate a practically needed quantity only to the precision which can be utilized in fulfilling the need. Also if a computation depends on certain parameters, and the errors in some parameters dominate the computation, then it is a common practice to ignore the other parameters whose inclusion in the computation does not influence the result in an appreciable way.

All procedures for determining the qibla suffer from several observational and numerical errors.

1. To begin with, geographical coordinates are determined by observations in which experimental errors are unavoidable. Moreover, the exact location whose latitude and longitude are used in the qibla computation is usually different from the location (e.g., a mosque) where the qibla is sought. The coordinates listed in gazetteers are for some survey station or observatory near a city, and may substantially differ from the exact coordinates of the desired location.

2. Apart from the errors involved in the input quantities (the coordinates), the computational methods themselves may introduce numerical errors. Nowadays, this is not very serious, since it is possible to use formulas which are simple and involve very few computational steps.

3. If the qibla method involves finding the north direction, then there are bound to be experimental errors in determining the north as well as in measuring the computed angle from it to determine the qibla line.

4. Small angular deviations from the exact qibla direction are unavoidable in the physical construction of mosques and in the marking of prayer rows.

5. Finally, it is physically impossible for a person who is praying to maintain the qibla orientation perfectly.

The last factor alone can easily introduce a deviation on the order of 10°. Some factors are less critical; for example, a site survey can provide geographical coordinates that are accurate to within a fraction of a degree. Other factors depend on the quality of engineering instruments and processes. But the accumulated error from all factors (ignoring 5, of course) can easily amount to about 5°, as a rough estimate. In view of so many sources of practical errors, it makes little sense to add the minor correction due to the earth's oblateness.

Incidentally, Kyrala[17] gives a formulation, although not a solution, of the equation for the direction of qibla as a geodesic on a general surface. It is not the ignorance of Muslim scientists but the lack of practical motivation why no serious investigation of the qibla problem on the geoid has been undertaken.

The claim that Muslim scholars knew about the oblate shape of the earth before western astronomers did is astounding. Muslim scholars have other contributions to geodesy to their credit, such as an accurate estimation of the circumference of the earth during the reign of al-Mamun (786–833). But I am not aware of any other reference that the Muslim scholars knew that the earth was oblate, prior to this becoming well-known late in the 17th century from the work of European scientists. On p. 24, N&K list the following scholars among those that "knew precisely that the shape of Earth was oblate:" al-Ramli (d. 1004 AH), al-Dusuqi (d. 1230 AH), 'Ulaysh (d. 1299 AH), al-Buhuti (d. 1046 AH), and Ibn 'Abidin (d. 1253 AH). It is not surprising that the scholars living in 1200 AH (1785 AD) and later knew about the earth's oblateness; but exact citations dating back to around 1000 AH (1591 AD) would be very valuable.

A.7

On p. 22, N&K state:

> Actual observation shows that the compass that *ˆAbdullah al-ˆAbdaliyy* and his followers depend on points to the wrong direction of al-Qiblah when used in the mosques of the Messenger of *Allah* in *al-Madinah al-Munawwarah.* Knowing this, should one follow this compass and the triangular calculations? . . . Likewise is the case in the Umayyad Mosque in Damascus. . . What does *al-ˆAbdaliyy* and his followers want us to do?

The remark about the Prophet's Mosque presumably refers to al-Muqrizi's report (see, for example [5, p. 83]) that, when measured during Ibn Touloun's times, the orientation of this mosque was found to differ by 10° from the qibla computed according to the then known values of geographical coordinates. The computed value of qibla depends on the algorithm used and the values of geographical coordinates that are obtained by experimental observation. In the past, it was also influenced much by the computational techniques used, accuracy of available tables, etc. It is not clear what values of coordinates were used and what computational method was followed at the time of Ibn Touloun.

N&K's criticism is misdirected, and shows a total misunderstanding of both methods of qibla determination—the one they themselves advocate and the great circle method that they oppose. Contrary to what N&K want to prove, the analysis of the qibla at Medina shows, in fact, the flaw of *their* approach. If Medina is exactly to the north of Mecca, then both methods give identical results, and have the same variance with the actual orientation of the Prophet's Mosque. If Medina is north as well as slightly east or west of Mecca, then the great circle definition comes much closer to the actual orientation than the N&K's method. This will now be explained.

A rule that N&K follow is that if a location is to the north of Mecca, then its qibla is to the south. But this is precisely what the great circle definition implies also. A location to the north of Mecca is on Mecca's meridian which, like all meridians, is a great circle. Therefore, the great circle direction is along the meridian itself. The "triangular calculation" in this case is trivial, like adding zero to a number or multiplying a number by one. But if the calculation is performed anyway, then the answer for a location north of Mecca will be the qibla bearing of 180°, that is, due south.

If I understand correctly, N&K's determination of the qibla depends only on *whether* a location is east or west and north or south of Mecca, not on *how much* east or west and north or south it is with respect to Mecca. The latter, of course, is the information that latitudes and longitudes contain, not utilized by N&K since in their opinion "religious judgments are not based on the instruments of astronomy and engineering." Now for a location not exactly north of Mecca, N&K[21, p. 16]quote this from al-Muqrizi: ". . . for he who is between north and west from *al-Kaˆbah*, the direction of his *Qiblah* is between south and east." This means that if a location is both north and west of Mecca, no matter how slight the western orientation (as long as it can be sensed somehow), the qibla at that location will be to the southeast (135° bearing) according to N&K. Similarly, for a location north and slightly east of Mecca, the qibla according to them is southwest. I have seen different values of Medina's coordinates in different gazetteers, with its longitude ranging from slightly east to slightly west of Mecca's. For example, using the coordinates ($24^\circ 26'$ N, $39^\circ 42'$E) for Medina and ($21^\circ 25'$ N, $39^\circ 49'$E) for Mecca, the resulting qibla bearing is 178°, that is 2° E of S. The great circle direction computed from other values range about $\pm 3^\circ$ from due south. These are almost due south and seem very reasonable. Using N&K's prescription, the direction would be southeast or southwest, that is 45° E or W of S.

In N&K's method, the qibla depends only on relative east/west and north/south orientation. But the rhumb line direction depends, of course, on the values of latitudes and longitudes. The great circle and rhumb line directions are very close for short distances because a small region of the earth behaves like a flat surface. The two directions start diverging as the distances increase and the earth's curvature becomes prominent. For locations close to Mecca, say within 2000 kilometers from it, the great circle direction and the rhumb line direction are practically the same. For Damascus, both methods give the same bearing 165°, that is 15° E of S. (N&K's method, of course, gives 45° E of S, since Damascus is both north and west of Mecca.) Clearly, it is incorrect to single out the great circle definition for any variation between the computed qibla and the actual orientation of the Omayyad Mosque.

As for the commercially available "qibla compasses," they vary a great deal in accuracy and the computation used in their manufacture. Certainly not all of them are correct because on p. 24 N&K themselves mention that some compasses show the qibla for North America to the southeast.

To drag compasses into the discussion also shows a very confused understanding of the issue. The "triangular calculation," (meaning the great circle qibla computation), does not depend on any compass; it just produces an angle with respect to north. In the case of Damascus, using the above coordinates for example, the angle produced by the great circle formula is about 15° E of S, just as the angle produced by the method N&K advocate would be 45° E of S. In each case, it still remains to find the north direction. The compass is one means of solving that separate problem. A (magnetic) compass gives the magnetic north direction, from which one can first obtain the true north by turning through the magnetic declination angle, and then the qibla by additionally turning through the qibla angle. It is not necessary to use a magnetic compass to determine the north, as there are several astronomical observations to do that more accurately. One preferable alternative to using the compass is observing the direction of Polaris, as described by N&K themselves.[19] Moreover, there are methods (see, for example, [1, pp. 290–292]) that can be used to determine the qibla directly from shadows, without needing to know the north direction at all.

[19] For an accurate determination of the north direction, the observation of Polaris should be performed at the time of Polaris' meridian passage, which can be found in almanacs.

A.8

On p. 11, N&K conclude:

> …determining the direction of *al-Qiblah* relies on knowing the position of the country from Makkah based on the sun, moon, stars, and the like. As you can see, it is not mentioned in any text that the reliance is on the shortest distance, or on the statement of mathematical calculations, or that of spherical triangles as some intelligence-claimers voice today.

Then on p. 14 they state:

> …not any [jurisprudents] have ever considered the shortness or the length of the route, nor the proximity or farness from Makkah in determining *al-Qiblah*.

N&K do not seem to realize that the route length considerations are implicit in those scholars' opinions. We cannot say that a location is east or west of another location unless we take into consideration the length of the routes between them.[20] What do we mean when we say that the location A lies to the east of the location B? Do we mean just that we can reach A by traveling continuously in a generally eastern direction (that is, allowing northern or southern, but not western, excursions) starting from B? No, we mean something more: that the shorter distance from B to A is in the eastern direction. If we do not insist on the shorter distance, then A is east as well as west of B, for we can surely also reach A by traveling west from B across the globe. To be precise, the relation between two points of being relatively east or west of each other is in the direction in which the absolute difference of their longitudes is less than 180°.[21]

If distances are to be ignored, then we might as well say that Mecca is to the west of North America. We consider Mecca as being to the east, rather than to the west, of (most of) North America solely because it is shorter to reach Mecca from North America by traveling eastward than westward. Incidentally, for a small part of North America that includes most of Alaska, Mecca is, indeed, to the west. From Anchorage, Alaska, for example, Mecca is to the west, since even the shortest eastern route requires a longitude traversal of more than 180°.

On the map given on p. 17 in N&K's book, Alaska appears to the west of Mecca and Japan appears to the east of USA. Both are illusions. Alaska lies east of Mecca and Japan lies west of USA since the shorter routes determine so. Flat maps of the world can be deceptive even about east and west! The statements in [21] by Sadek (p. 63), Klinkenberg (p. 66), and Jance (p. 67), declaring North America to be northwest of Mecca or Mecca to be southeast of North America are imprecise in not making the exception for Alaska. We have seen that between any two points on the earth which have different longitudes, there exist an infinite number of rhumb lines, going both eastward and westward. So to define the direction even by rhumb lines, we have to consider the shortest route. In the rhumb line sense, Mecca is southwest of nearly all of Alaska.

The same objection applies to the religious rulings displayed in [21, English versions on pp. 37, 40, 42, 44, 45, 47, 49–50, 52]. Some of them (pp. 40, 45, 50) go to the extent of explicitly declaring that *North America is in the northwestern quarter with respect to Mecca*. Since they do not mention an exception for Alaska, one would infer that they consider Alaska to be also northwest of Mecca. If this is indeed the case, then—whatever the religious merit of those rulings may be—their geographical statement is false.

[20] We can define the east, west, north, and south directions at any single location without involving the concept of distance. We can also determine whether a location is north or south of another location without knowing their distance. But in determining whether a location is east or west of another, we have no choice but to take into consideration "the shortness or the length of the route."

[21] There are other ways to determine whether A is east or west of B. Due to the earth's daily rotation, phenomena such as culmination of heavenly bodies are seen first in the east then in the west. But since these phenomena recur, something which first happened at A then at B will then again happen at A. So the occurrence at B is both before and after the occurrence at A. We say that A is east of B if the occurrence at A is less than 12 hours earlier than at B. This is equivalent to saying that we reach A by traveling east from B while traversing less than 180° in longitude.

Their conclusion is derived from a Mercator projection map (such as on [21, p. 17]) which is longitudinally centered near Greenwich, England, not on Mecca. The quarters with respect to Mecca are more correctly shown in the Mercator projection centered on Mecca given in the map in Figure 8.

A.9

On p. 15, N&K paraphrase al-Ghazali as follows: “it is well known that north is opposite to south and that east is opposite to west and their lines meet at right angles with each other.” This statement is to be interpreted carefully for a round earth. North-south and east-west relationships are not absolutely similar, and the “opposition” between north and south is different from that between east and west. On the earth, there are two distinct points, namely the north and south poles, that can be considered most north and most south of every given location. But there are no correspondingly extreme east and west points on the earth. For any location, the most eastern and most western points on its latitude coincide. $180°$ East or $180°$ West is the same longitude.

At the north pole, every direction is south, and at the south pole every direction is north. The quoted statement does not hold at the poles. The statement that the east-west (“parallels”) and north-south lines (“meridians”) intersect at right angles does not apply to the poles, and elsewhere it is about spherical angles and local relationships. If two systems of lines on a plane intersect at right angles, then the lines in each set are parallel. On a sphere, this is not so. The north-south lines are not parallel, but converge on the poles.

A.10

In several places, N&K quote scholars to the effect that Polaris is among the “strongest signs.” N&K have taken this to mean that Polaris is fixed and exactly north. So after mentioning the saying of several scholars, N&K say on p. 15:

> Hence, the direction of Polaris is north. The scholars mentioned that *Suhail* is another star that always points south. . . . East is known, and it is the direction from where sun rises, and west is where the sun sets.

I would not have taken issue with these statements had they been presented as simple, approximate, and practical rules of thumb—as intended by the quoted scholars. But since they have been presented by N&K as absolute truths, it is necessary to point out their errors.

1. Polaris is not exactly on the earth’s axis of rotation, but off it by a small amount which is slowly changing and is about $47'$ these days. Therefore it is not fixed in the sky, but moves in a small circle. Many popular astronomy books have long-exposure night photographs of the circumpolar region of the sky, showing the trail of Polaris as a bright arc. This star points in exactly the north direction only twice during each 24-hour period; these times are listed in many almanacs as the “meridian crossing times of Polaris.”

2. The star *Suhail* or Canopus is not a reliable indicator of south at all as it is too far ($37°$) from the celestial south pole. Its connection with south was that in Mecca’s vicinity its rising point was very nearly south. The celestial south pole is not visible from the northern hemisphere. Had Canopus been at the south pole, it would not have been visible from the middle eastern region at all, and would not have been mentioned by early Muslim scholars.

3. Only on two days during the year, namely during the spring and fall equinoxes which occur approximately on March 21 and September 21, does the sun rise from the east and set in the west. From the

> spring equinox to the fall equinoxes, the sun rises from a northeastern direction and sets in a northwestern direction. Then from the fall equinox to the spring equinox, the sun rises from a southeastern direction and sets in a southwestern direction. The deviation, north or south of the east-west direction, first progressively increases each day, then progressively decreases each day. The maximum deviation takes place on the days of summer and winter solstices, approximately June 21 and December 21. The deviation, north or south, also depends on the latitude, being more pronounced at higher latitudes. At latitudes close to 66° N, both sunrise and sunset take place in practically the north direction during high summer days. At latitudes higher than $66\frac{1}{2}^\circ$ N, the sun does not rise or set for several days in the summer, but just circles in the sky, once every 24 hours. The sun's period of continuous visibility gets longer with higher latitudes, until at the north pole the sun never rises or sets during the entire time from the spring to the fall equinox. We have described the phenomena as they occur in the northern hemisphere; similar phenomena occur in the southern hemisphere but in opposite seasons.

Another scientifically objectionable statement is on p. 19:

> Moreover, the sun runs a course that we do not know.

The sun's movement in the sky is understood as well as, or better than, that of any other heavenly body. At no point during any day, does the sun become unaccountable, since it is visible in the sky continuously for six months either from the north pole or from the south pole. The opinion that the sun's whereabouts are unknown at night is contradicted by simple observation and belongs to prehistoric times.

A.11

On p. 19, N&K say:

> Endeavoring to determine *al-Qiblah* by the layman must be an easy matter free of hardships. It is sufficient for him to look at the direction of sunrise to know the four directions and hence to determine his position from *Makkah*. Also he would look at Polaris or star *Suhail* to know his proximity or farness from *Makkah* northward or southward. This, as you can see, does not require complex calculations nor a dive into an ocean of geometry and mathematics to determine the direction of *al-Qiblah*.

In N&K's opinion, every one has the obligation to find the qibla independently as (p. 18) "a seeing person is not entitled to follow somebody else in the direction of *al-Qiblah*, because he can endeavor to determine the direction himself." So they have given above a procedure that they believe any one can use to determine the qibla easily.

At the north and south poles, there are no east or west directions. Hence N&K's procedure for determining the qibla is clearly incomplete. But let us look at what it entails for other locations.

One is first supposed to determine whether one's location is to the north or south of Mecca by looking at Polaris (or Canopus) in both places. A moment of reflection shows that this method is difficult and expensive, and often useless. To begin with, this necessitates a journey to Mecca. Moreover, since N&K would not have one rely on instruments to record the altitudes, and since the observations to be compared are likely to be several days apart (many months apart in the past), one needs an extraordinary memory of the observations. Finally, the observations would often be inconclusive because a small altitude difference (say, up to 5–8 degrees) is difficult to discern visually.

It is unimaginable that people in Medina used naked-eye Polaris observations to ascertain that they were to the north of Mecca, because Polaris would appear to be almost at the same height in Medina as in Mecca. (Their latitudes, and hence Polaris' altitudes in these locations, differ only by 3° degrees.) Similarly, Polaris

observations, unaided by instruments, will scarcely be able to resolve whether Houston, Miami, Guatemala City, Jamaica Island, Dakar, Khartoum, Karachi, and Manila are north or south of Mecca.

Whether a location is relatively north or south of Mecca is only half the information needed by N&K in determining the qibla. Nowhere have they specified a method for the other half—whether a location is relatively east or west of Mecca. Looking at the direction of sunrise is, for most days of the year, an inaccurate solution of the trivial problem of determining the east direction. By simply looking at the sun or the stars in a location, one cannot tell whether that location is to the east or west of Mecca.

What N&K really rely on—without admitting—are plane geographical maps, such as the one they give on p. 17. As we have seen, the construction of these maps requires the latitude and longitude information as well as map projection equations, and is thus dependent on both scientific instruments and trigonometric calculations.

A.12

The response from Bareilly (p. 58) in Arabic has been reproduced without an English version. Its relevant part can be translated as:

> 1. [Of] the circle of altitude passing through the zeniths of Mecca and Washington, the smaller arc lies in the eastern direction from Washington when facing the Ka‘ba. Hence, the Ka‘ba is to the east.
>
> 2. And the latitude of the location [Mecca] is less than the latitude of Washington . . . . Therefore, the qibla angle is southern, as discussed in the science that investigates the derivation of the direction of qibla.

This response betrays the lamentable fact that although in many subcontinental Islamic colleges classical texts such as *Sharh Chaghmini* are still being taught, their teaching has now diminished to just reciting the words. The definition of qibla has been quoted here correctly but, I suspect, without understanding what it means. The rest, unrelated to that definition, is an attempt to derive a conclusion which agrees with a ruling from al-Azhar. It is saddening to see this rambling, meaningless statement issued officially as a religious ruling from such a distinguished institution as Bareilly.

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